

**Artificial Neural Network for Real
Time Load Flow Calculation:
Application to a Micro Grid with
Wind Generators**

This work presents a method for solving the problem of load flow in electric power systems including a wind power station with asynchronous generators. For this type of power station, the generated active power is only known and consequently the absorbed reactive power must be determined. So we have used the circular diagram at each iteration and by considering this node as a consuming node in the load flow program. Since the wind speed is not constant, the generated power is neither constant. To predict the state of the network in real time, we have used the artificial neural networks after a stage of training using a rich base of data.

Keywords: Electrical supply networks, load flow, microgrid, circular power diagram, wind generator, artificial neural networks.

1. INTRODUCTION

The crisis energy due to the increase of the fuel cost is one of the principal factors, which motivates the companies of production, transport and distribution of the electricity to plan the development of renewable energies based power stations [1-4]. These last years the wind power stations experienced a great development making it possible to offer an increasingly high power unit with production costs, which become increasingly competitive in comparison with conventional sources. Moreover, the technical and economic reasons and the environmental aspect justify the development of this type of production. Indeed, policies of the states promote clean technologies in order to reduce the emissions of carbon dioxide and encourage renewable energies by subsidies and dedicated tariffs, which lead to interesting economic conditions.

The integration of this type of power stations in an electrical network requires a different study from the traditional cases with power stations, which are based on synchronous machines. Indeed, different technologies of machines are used in wind power stations and complicate the model [5].

In this paper we study the problem of load flow in an isolated medium voltage (MV) grid ring, which can operate in islanding mode. This network includes traditional power stations and, in addition, a squirrel induction machine based

wind power station. The co existence of various machine technologies requires a dedicated input/output orientation of electrical quantities and so a particular formulation of power equations.

The load flow calculation consists in determining the state of the network for a given wind speed [6]. For this task, the wind speed is sensed and used to estimate the maximum active power by using the characteristics of the wind turbine and the induction machine. Then, the power circular diagram gives the necessary reactive power and then classical calculation methods can be used.

Since the wind speed is not constant, the power generated by the wind park is not constant. A data base is then created with different wind speeds and corresponding results from the load flow problem. Thus, we establish a training algorithm whose inputs are the wind speed and the total required power and outputs are the voltages in modulus and phases at the various nodes. We can deduce the other variables such as the reactive power, which is produced by the various machines and the active and reactive losses of the network. Thus, with the application of the artificial neural networks, we can obtain the various solutions of the problem in real time and according to a forecast curve of the requested total power and a forecast curve of the wind.

2. EQUATIONS OF THE ELECTRIC POWER NETWORK

2.1 Equations of the powers

A branch of a network between two nodes i and j can be modeled by five admittances [7,8] (figure 1):

\bar{Y}_{ii}^m is the admittance of the elements which are connected between node i and ground (transmitters, testing devices, bench of coil or of capacity \bar{O}).

\bar{Y}_{ij}^s is the admittance of the half capacity of the line ij .

\bar{Y}_{ij}^l is the admittance of the line ij .

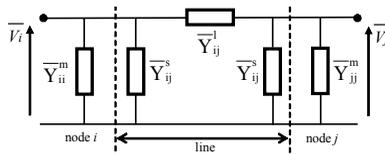


Fig. 1. Model of a branch

A node i can be connected to several nodes. The current injected in node i is written by:

$$\bar{I}_i = \bar{Y}_{ii}^m \bar{V}_i + \sum_{j \neq i}^N \bar{Y}_{ij}^s \bar{V}_j + \sum_{j \neq i}^N \bar{Y}_{ij}^l (\bar{V}_i - \bar{V}_j)$$

where N is the number of nodes.

This current can be written in the following matrix form: $\bar{I} = \bar{Y}\bar{V}$

With:

$$\bar{Y} : \begin{cases} \bar{Y}_{ii} = \bar{Y}_{ii}^m + \sum_{j \neq i}^N (\bar{Y}_{ij}^l + \bar{Y}_{ij}^s) \\ \bar{Y}_{ij} = -\bar{Y}_{ij}^l \end{cases}$$

The injected powers respectively apparent, active and reactive, in any node of the network are given by:

$$\begin{cases} \bar{S}_i = P_i + jQ_i \\ P_i = \sum_{j=1}^N Y_{ij} V_i V_j \cos(\alpha_i - \alpha_j - \theta_{ij}) \\ Q_i = \sum_{j=1}^N Y_{ij} V_i V_j \sin(\alpha_i - \alpha_j - \theta_{ij}) \end{cases} \quad (1)$$

With Y_{ij} and θ_{ij} are respectively the modulus and argument of an admittance (i, j) of the nodal matrix, V_i and α_i are respectively the modulus and argument of the voltage at the node i .

In a general form these powers are written as

$$\begin{cases} P_i = P_{gi} - P_{ci} \\ Q_i = Q_{gi} - Q_{ci} \end{cases} \quad (2)$$

Where P_{gi} and Q_{gi} represent respectively the generated active and reactive power at the node i , P_{ci} and Q_{ci} represent respectively the consumed active and reactive power at the node i .

2.2 Modeling of the wind power station

1. Presentation of the wind generator

Many technologies of wind generators exist. We consider here an Induction Machine Based Wind Generator (IMBWG). The squirrel cage induction

generator is driven by a horizontal axis wind turbine and is directly connected to the power network (figure 2).

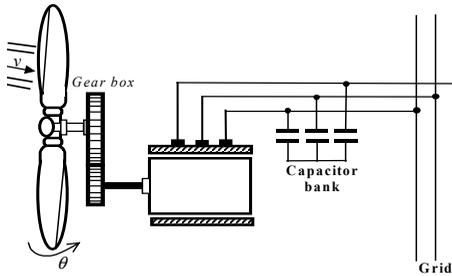


Fig.2. Wind turbine system configuration

The short-circuited asynchronous rotor machine cannot operate like an autonomous generator. It requires reactive power from the grid for its own excitation. It is necessary to connect a capacitor bank for reactive power compensation. These generators are also characterized by a bad power-factor, because the grid must ensure the magnetization of the machine, in other words the installation of the coupling magnetic field in the air-gap. Actually, the majority of the wind generators are still based on an induction machine with direct connection to the grid (75 % to 90 % according to the literature). The studied park consists of 10 identical aero generators whose total rated power is 7.5 MW.

2. Modeling of the induction machine

To model the induction machine, one uses an equivalent electrical diagram single-phase condition. It is supposed that the Kapp assumption is valid. This amount neglecting the voltage drop caused by the current magnetizing in the resistance and the reactance of the stator leakages. Under these conditions, a phase of the stator is depicted on the figure 2.

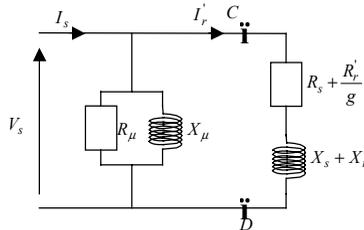


Fig. 3. Equivalent electrical circuit of the induction machine

Where R_s and X_s are respectively the resistance and the leakage reactance of the stator, R_r' and X_r' are respectively the resistance and the leakage

reactance of the rotor brought back to the stator, R_{μ} is the equivalent resistance corresponding to the iron losses, X_{μ} is the magnetizing reactance of the stator, and g is the slip.

While considering $R = R_s + \frac{R_r'}{g}$ et $X = X_s + X_r'$, the active and reactive power seen between points A and B are expressed as:

$$\begin{aligned} P' &= V_s^2 G \\ Q' &= V_s^2 B \end{aligned} \tag{3}$$

G and B represent respectively the conductance and the susceptance of the equivalent admittance of the circuit seen between the points C and D (see the annex).

The variation of the reactive power Q_i according to the active power P_i is given by the following power circle equation (see the annex):

$$\left(Q' - \frac{V_s^2}{2X} \right)^2 + P'^2 = \left(\frac{V_s^2}{2X} \right)^2 \tag{4}$$

The active power which is dissipated in R_{μ} (P_f) and the reactive power which is consumed by X_{μ} (Q_f) are constant since the voltage is assumed constant. Then the representation of the power diagram (P, Q) of the induction machine is derived from the previous power diagram $P' Q'$ with a change of the reference mark:

$$\left(Q - \frac{V_s^2}{2X} \right)^2 + P^2 = \left(\frac{V_s^2}{2X} \right)^2 \tag{5}$$

with: $Q = Q' + Q_f$ and $P = P' + P_f$

The capacitor bank enables to obtain a zero reactive power at the connection point.

3. LOAD FLOW PROBLEM

3.1 Presentation

The problem of load flow in an electrical supply network consists in determining its operating condition in permanent mode; i.e. to determine the voltages in modulus and argument at the various nodes. The other electrical

quantities such as the injected powers, the power flow, and the losses... can be deduced. The resolution of this problem assumes a correct definition of data and unknown quantities at the various nodes.

Synchronous Machine Based Generators (SMBG) masters the generated power (P_g) and the voltage (V) according to the consumed active and reactive power (P_c and Q_c). The generated active power of IMBWG (P_g) may be estimated by sensing the wind speed. Active and reactive powers of loads (P_c , Q_c) are known. The node assessment is selected among the SMBG nodes.

Table I specifies, for each type of node, the data and the unknown factors.

TABLE I: Data and unknown quantities at the various types of nodes

Type	Nbr	Known	Unknown	Deductions
SMBG	N_{p-1}	P_g, P_c, Q_c, V	α	Q_g
IMBWG	N_e	P_g, P_c	Q_c, V, α	Q_g
Load	N_c	P_c, Q_c	V, α	-
Assessment	I	V, α, P_c, Q_c	-	P_g, Q_g

3.2. Formulation of the load flow problem

The state of the network must consider the IMBWG and so is determined while passing by two stages:

- a) the first stage consists in determining the reactive power of the induction machines for a given estimated active power and an initial value of the voltage.
- b) the second stage supposes that the wind power generators are load nodes (with then a negative flow of powers) and so consists in writing for each load nodes :
 - $N_{p-1} + N_e + N_c$ equations implying active powers (P equations) to determine the arguments of the voltages (α),
 - $N_e + N_c$ equations implying reactive powers (Q equations) to determine the modules of the voltages.

The stage (b) can be written as follows:

$$\begin{cases} \underline{F}_p(\underline{\alpha}, \underline{V}) = \underline{0} \\ \underline{F}_q(\underline{\alpha}, \underline{V}) = \underline{0} \end{cases} \quad (6)$$

$$\text{With: } \begin{cases} F_{pi} = -P_{gi} + P_{ci} + \sum_{k=1}^N Y_{ik} V_i V_k \cos(\alpha_i - \alpha_k - \theta_{ik}) \\ i = 1, \dots, N_c + N_e + N_{p-1} \\ F_{qi} = Q_{ci} + \sum_{k=1}^N Y_{ik} V_i V_k \sin(\alpha_i - \alpha_k - \theta_{ik}) \\ i = 1, \dots, N_c + N_e \end{cases}$$

3.3 Resolution of the load flow problem

The resolution of this problem uses iterative methods [9,11,12] since it is about a non-linear problem. The Newton-Raphson method [9,12] constitutes the universal method for the resolution of this problem. This method is based on the successive evaluation of the Jacobean matrix.

Variations of the functions F_{pi} and F_{qi} between two points $\underline{X}_0 = \begin{bmatrix} \alpha_0 \\ V_0 \end{bmatrix}$ and

$\underline{X}_1 = \begin{bmatrix} \alpha_1 \\ V_1 \end{bmatrix}$ result in:

$$\begin{cases} F_{pi}(\underline{X}_1) = F_{pi}(\underline{X}_0) + \nabla_{1i}^T(\underline{X}_0) (\underline{X}_1 - \underline{X}_0) \\ F_{qi}(\underline{X}_1) = F_{qi}(\underline{X}_0) + \nabla_{2i}^T(\underline{X}_0) (\underline{X}_1 - \underline{X}_0) \end{cases}$$

With:

$$\begin{cases} \underline{\nabla}_{1i} = \left(\frac{\partial F_{pi}}{\partial \alpha_1} \quad \frac{\partial F_{pi}}{\partial \alpha_2} \quad \dots \quad \frac{\partial F_{pi}}{\partial \alpha_{n-1}} \quad \frac{\partial F_{pi}}{\partial V_1} \quad \frac{\partial F_{pi}}{\partial V_2} \quad \dots \quad \frac{\partial F_{pi}}{\partial \alpha_{Nc}} \right)^T \\ \underline{\nabla}_{2i} = \left(\frac{\partial F_{qi}}{\partial \alpha_1} \quad \frac{\partial F_{qi}}{\partial \alpha_2} \quad \dots \quad \frac{\partial F_{qi}}{\partial \alpha_{n-1}} \quad \frac{\partial F_{qi}}{\partial V_1} \quad \frac{\partial F_{qi}}{\partial V_2} \quad \dots \quad \frac{\partial F_{qi}}{\partial \alpha_{Nc}} \right)^T \end{cases}$$

The system can be expressed in a matrix form:

$$\begin{bmatrix} \underline{F}_p(\underline{X}_1) \\ \underline{F}_q(\underline{X}_1) \end{bmatrix} = \begin{bmatrix} \underline{F}_p(\underline{X}_0) \\ \underline{F}_q(\underline{X}_0) \end{bmatrix} + \begin{bmatrix} J_{p\alpha} & J_{pv} \\ J_{q\alpha} & J_{qv} \end{bmatrix}_{(\underline{X}_0)} [\underline{X}_1 - \underline{X}_0] \quad (7)$$

$$\text{With: } \begin{cases} J_{p\alpha} = \frac{\partial \underline{F}_p}{\partial \underline{\alpha}} ; J_{pv} = \frac{\partial \underline{F}_p}{\partial \underline{v}} \\ J_{q\alpha} = \frac{\partial \underline{F}_q}{\partial \underline{\alpha}} ; J_{qv} = \frac{\partial \underline{F}_q}{\partial \underline{v}} \end{cases}$$

The objective of the passage from an item \underline{X}_0 to another item \underline{X}_1 is to make tighten vectors $\underline{F}_p(\underline{X})$ and $\underline{F}_q(\underline{X})$ towards origin $\underline{0}$. This passage is conditioned by:

$$[\underline{X}_1] = \begin{bmatrix} \underline{\alpha}_1 \\ \underline{V}_1 \end{bmatrix} = \begin{bmatrix} \underline{\alpha}_0 \\ \underline{V}_0 \end{bmatrix} - \begin{bmatrix} J_{p\alpha} & J_{pv} \\ J_{q\alpha} & J_{qv} \end{bmatrix}^{-1} \begin{bmatrix} \underline{F}_p(\underline{\alpha}_0, \underline{V}_0) \\ \underline{F}_q(\underline{\alpha}_0, \underline{V}_0) \end{bmatrix} \quad (8)$$

In the case of the electrical supply network, one generally checks that the active power is very sensitive to the angle whereas the reactive power is very sensitive to the voltage. By adopting this assumption, the relation (8) is reduced to:

$$\begin{cases} \underline{\alpha}_1 = \underline{\alpha}_0 - J_{p\alpha}^{-1} \underline{F}_p(\underline{\alpha}_0, \underline{V}_0) \\ \underline{V}_1 = \underline{V}_0 - J_{qv}^{-1} \underline{F}_q(\underline{\alpha}_0, \underline{V}_0) \end{cases} \quad (9)$$

One will speak about the uncoupled load flow. The elements of matrices $J_{p\alpha}$ and J_{qv} are expressed by:

$$J_{p\alpha} = \begin{cases} (i \neq j) : V_i V_j Y_{ij} \sin(\alpha_i - \alpha_j - \theta_{ij}) \\ (i = j) : -\sum_{i \neq j}^N V_i V_j Y_{ij} \sin(\alpha_i - \alpha_j - \theta_{ij}) \end{cases} \quad (10)$$

$$\text{and } J_{qv} = \begin{cases} (i \neq j) : V_i Y_{ij} \sin(\alpha_i - \alpha_j - \theta_{ij}) \\ (i = j) : -V_i Y_{ii} \sin(\theta_{ii}) + \frac{Q_i}{V_i} \end{cases} \quad (11)$$

3.4 Algorithm

We defined a participation factor written F_c in order to characterize the distribution of the requested total power P_D on the various nodes. Another distribution factor noted F_g , which characterizes the distribution of the total generated power at the various producing nodes is defined. Each node is connected to a load where its power-factor $\cos \phi_c$ is assumed to be constant. So, we have:

$$\begin{aligned} \underline{P}_g &= \underline{F}_g P_D \\ \underline{P}_c &= \underline{F}_c P_D \\ Q_{ci} &= P_{ci} \operatorname{tg}(\phi_{ci}) \end{aligned} \quad (12)$$

By using these notations, the load flow algorithm is as follows:

1. reading the data and writing the nodal matrix,
2. initializing $\underline{\alpha}_0, \underline{V}_0, P_D$, the wind speed and the tangent of the angle $\tan \phi_c$,
3. calculating the reactive power of the wind mills and suppose the nodes where they are as being consuming nodes,
4. calculating the functions \underline{F}_p and \underline{F}_q ,
5. building the Jacobean matrixes $J_{p\alpha}$ and J_{qv} ,
6. calculating the angles and the voltages by :

$$\begin{cases} \underline{\alpha} = \underline{\alpha}_0 - J_{p\alpha}^{-1} \underline{F}_p \\ \underline{V} = \underline{V}_0 - J_{qv}^{-1} \underline{F}_q \end{cases} \text{ and then return to the stage 3.}$$

The iterations must be stopped as soon as the functions \underline{F}_p and \underline{F}_q become lower than a wished value, or when these functions remain constant.

4. USE OF AN ARTIFICIAL NEURAL NETWORK

4.1 Techniques of training

Artificial neural networks (ANN) are mathematical models, which are made up of non-linear calculating units. These units are connected each other through weights whose values are adaptive parameters which are adapted to map a mathematical function. ANN have shown a great aptitude to confront various types of complex modeling problems [10,13,15,16]. They have the capacity to learn by adapting their parameters. Weights are calculated by iterative training algorithm described by mathematical rules. The most often used architecture is the structure multi-layer [10,15]. The algorithm of back propagation of the error [14] is the most known algorithm for the training of the multi-layer networks.

In this study, we used the Neural Network Toolbox from the Matlab environment. The structure of the used ANN has two input units, one hidden layer and one output layer with eight units (figure 5).

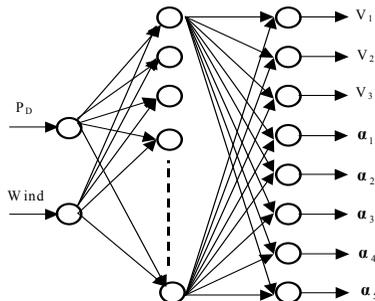


Fig.5. Structure of the ANN

The data base is made by using 1600 values, which correspond to the combination of 40 values of the consumed power and 40 values of the wind speed (figures 2 and 3).

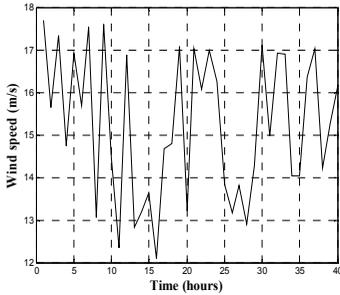


Fig. 2. Wind speed used for the training algorithm

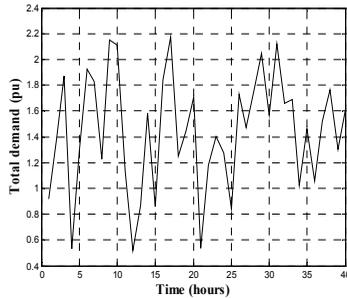


Fig. 3. Total consumed power for the training algorithm

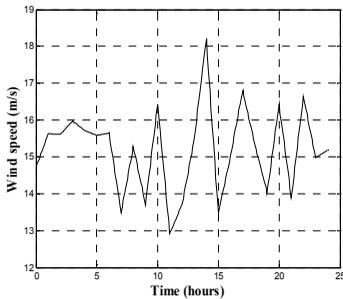


Fig. 4. Forecast wind speed.

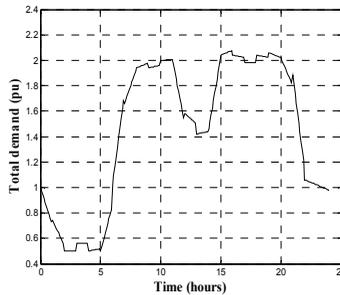


Fig. 5. Forecast total consumed power.

4.2 Forecast curves

We used the forecast curves of the requested total active power and the wind speed during one 24 hours period. These curves are represented on figures 4-5.

4.3 Presentation of the studied micro grid

The simulated micro grid is composed of 6 nodes, 4 generators and 7 lines. The generator *GI* is the studied wind power station (figure 6).

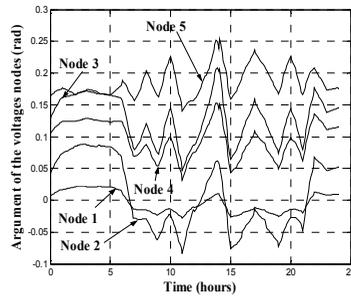
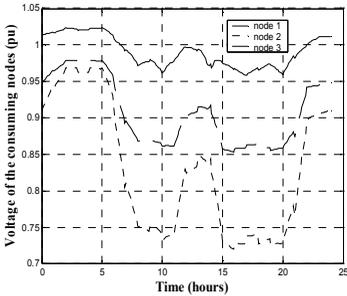


Fig. 7. Voltage of the consuming nodes. Fig. 8. Argument of the voltage nodes.

4.5 Comparison of neural network and traditional method

A comparison of obtained results with the ANN and with the traditional method is performed for various operating points, which are defined by the required active power PD, and the wind speed. Some results are shown in two tables V and VI.

The index c and n indicate respectively the obtained results from the traditional method and from the ANN. Tables V and VI prove well that the used ANN gives results, which coincide accurately with those from the traditional method. Indeed, the error is always related to the third term after the comma.

TABLE V: Voltages and angular speed comparison

Wind (m/s)	PD (pu)	V_{1c}/V_{1n} (pu)	V_{2c}/V_{2n} (pu)	V_{3c}/V_{3n} (pu)	Ω_c/Ω_n (rad/s)
13	1	0.9468	0.9112	1.0187	315.6303
		0.9470	0.9114	1.0187	315.6239
12	1.1	0.9393	0.8976	1.0162	315.2664
		0.9383	0.8948	1.0155	315.2662
14	0.9	0.9538	0.9237	1.0197	315.9066
		0.9540	0.9241	1.0198	315.9072
15	1.5	0.9093	0.8357	0.9942	316.2535
		0.9091	0.8354	0.9941	316.2511
16	2.3	0.8220	0.6463	0.9427	316.5222
		0.8275	0.6601	0.9458	316.5157

TABLE VI: Comparison of the obtained results argument

Wind (m/s)	PD (pu)	α_{1c}/α_{1n} (rad)	α_{2c}/α_{2n} (rad)	α_{3c}/α_{3n} (rad)	α_{4c}/α_{4n} (rad)	α_{5c}/α_{5n} (rad)
13	1	-0.0003	0.0155	0.0851	0.0747	0.1258
		-0.0004	0.0152	0.0846	0.0742	0.1251
12	1.1	-0.0076	-0.0102	0.0519	0.0517	0.1020
		-0.0079	-0.0129	0.0513	0.0489	0.1002
14	0.9	0.0057	0.0362	0.1110	0.0920	0.1427
		0.0058	0.0364	0.1111	0.0921	0.1427
15	1.5	-0.0035	0.0119	0.1261	0.1022	0.1885
		-0.0036	0.0117	0.1259	0.1021	0.1884
16	2.3	-0.0277	-0.0878	0.1199	0.0541	0.2091
		-0.0261	-0.0770	0.1219	0.0632	0.2130

5. CONCLUSION

The study of the load flow problem of electrical networks including wind power stations differs from the traditional case by the fact that the wind node is defined only by the active power. The voltage and the reactive power are unknown factors. We have used the circular power diagram to determine the reactive power and then to deduce the other unknown quantities. To approach the case of a real network, we have simulated the forecast curves of the wind speed and the total requested power. The traditional procedure takes along computing time and consequently the analysis of the results in real time is then impossible. To overcome this difficulty, we used an artificial neural network. The comparison of obtained results with the traditional method and with the ANN proves the effectiveness of the neuronal model for this type of problem.

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APPENDIX

After transforming the branch between the two points C and D into an equivalent parallel circuit, the admittance between these points is expressed as:

$$\bar{Y}_{CD} = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2} \quad (A1)$$

This can be rewritten in the following form:

$$\bar{Y}_{AB} = G - jB \quad (A2)$$

By identifying both preceding relations the real part yields to : $R = \frac{G}{B} \cdot X$

And the imaginary part yields to:

$$B^2 + G^2 - B/X = 0 \quad (A3)$$

This relation corresponds to a circle equation whose the center is $(1/(2X), 0)$

and the radius: $1/(2X)$:

$$(B - 1/(2X))^2 + G^2 = 1/(2X)^2 \quad (A4)$$

By multiplying the equation A4 by V_s^4 , we obtain the circular power diagram, which is described in (5).