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Coupled Analytical-Finite Element Methods for Linear Electromagnetic Actuator Analysis

In this paper, a linear electromagnetic actuator with moving parts is analyzed. The movement is considered through the modification of boundary conditions only using coupled analytical and finite element analysis. In order to evaluate the dynamic performance of the device, the coupling between electric, magnetic and mechanical phenomena is established. The displacement of the moving parts and the inductor current are determined when the device is supplied by capacitor discharge voltage.

Keywords Actuators, analytical analysis, coupling model, finite element method, movement, transient feeding.

1. INTRODUCTION

In electromagnetic devices, the electromagnetic field can be obtained by solving partial differential equations. The solution of these equations can be done through numerical methods such as finite element one [1]-[2]. Nevertheless in the case of dynamic studies of these devices, this method fail due to the flexion and the deformation of the solution domain subdivision when the mobile parts are moved. To solve this problem, generally a new mesh at each displacement step is required, and in this case, this method becomes cumbersome and very expensive. Several formulations are developed in order to take account of the movement in mobile systems such as the electric machines, the actuators, the induction heating systems, etc...

Generally, the existing formulations, allowing the movement simulation, make use of special elements or meshing modifications and lead to costly models [1]-[2]. An other approach is used for movement consideration in linear electromagnetic systems. This approach is based on only one finite element meshing for all the displacement steps as described in [3]. The major defect of this technique is the fact to simulate the movement to discontinuous steps. This led to numerical noises in calculation if finite element mesh is not well refined on the level of the movement zone.

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To overcome this problem, a coupled model based an analytical and finite element solution is proposed in this paper. In this model, the movement is taken account through the modification of boundary conditions only. The analytical solution is determined in a simple shape region considered between a moving part and a fixed one, and called «MZ : Movement Zone». The rest of the domain is finite element meshed. The analytical solution is coupled to the numerical one through the continuity condition for the field tangential component (Ht). The movement is then considered by only the modification of the points coordinates of the interface between both analytical and numerical sub-domains. In this way, the matrix due to the finite element discretization is calculated once and used for every relative position of the moving part. The formulation is elaborated in the case of an axisymmetrical structure and its validity is achieved when applying it to study an electromagnetic actuator.

In the other hand, the main design factors of the actuator : the displacement of the moving parts and the electrical current in the coil, are determined by the coupling between electric, magnetic and mechanical phenomena [4]-[7]. The problem is investigated by the parameterization coupling model. Measurements are carried out and compared to computed data when the device is supplied by capacitor discharge voltage.

2. FIELD EQUATIONS AND FORMULATION

Let consider an axisymmetrical electromagnetic system (Fig. 1). The load is a body which moves under the effect of the electromagnetic forces.

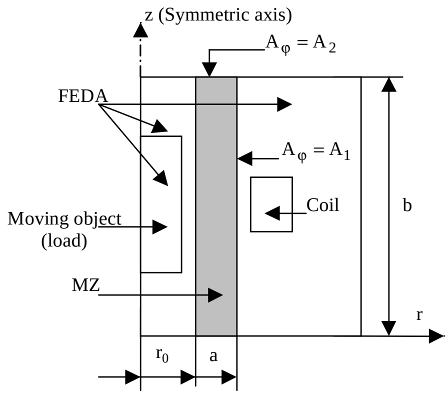

Fig. 1. Electromagnetic system objects. Note that : FEDA is the Finite Element Discretized Area and MZ is the Movement Zone.

In this case, the magnetic vector potential equations are as follows [3]:

$$\partial/\partial r\left((\nu_r/r).\partial\left(rA_\varphi\right)/\partial r\right)+(\partial/\partial z).\left((\nu_r/r)\partial\left(rA_\varphi\right)/\partial z\right) = -\mu_0\left[(J-(\sigma/r)(\partial V/\partial\varphi)\right] \tag{1}$$

$$J=-\sigma\left[\left(\partial A_\varphi/\partial t\right)+(\mathrm{v}x\mathbf{c}\mathrm{ur}l(\mathrm{A}))_\varphi\right]=-\sigma\left(DA_\varphi/\partial t\right) \tag{2}$$

A_φ and J are the components following the angular direction (φ). V : electrical potential, v : velocity, σ : electric conductivity, ν : magnetic reluctivity, μ_0 : vacuum permeability.

In MZ zone (Fig. 1), equation (1) becomes :

$$(\partial/\partial r)\ (1/r)\ \left[\partial\left(rA_\varphi\right)/\partial r\right]+(\partial/\partial z).(1/r)\left[\partial\left(rA_\varphi\right)/\partial z\right]=0 \tag{3}$$

The variables separation technique is applied to solve (3). When considering the following axisymmetrical conditions :

$$\begin{cases} A_\varphi(r_0+a,z)=A_\varphi(-r_0-a,z)=A_1 \\ A_\varphi\left(r,b/2\right)=A_\varphi\left(r,-b/2\right)=A_2=0 \end{cases} \tag{4}$$

the solution of this equation using Bessels functions is given as follows :

$$A_\varphi(r,z)=\sum_{k=0}^{\infty}\left\{\left[\left((-1)^k.4A_1\right)/\left((2k+1)\pi\right)\right] .\cos\left[\left((2k+1)(\pi/b)\right)z\right].\left[\left(J_1(r\lambda)\right)/\left(J_1\left[(r_0+a)\lambda\ \right]\ \right)\right]\right\} \tag{5}$$

with $\lambda=-j\left(2k+1\right)\left(\pi/b\right)$. J_1 is Bessel's function of first order and first kind.

In order to couple the analytical solution to the finite element analysis, one has to put the continuity condition in term of tangential component (H_t) of the magnetic field :

$$H_t=(\nu/r).\left[\partial\left(rA_\varphi\right)/\partial n\right] \tag{6}$$

To determine H_t, we calculate the derivation respect to the normal n :

$$
\begin{aligned}
\partial\left(rA_\varphi\right)/\partial n &= \left[A_\varphi + r\left(\partial A_\varphi/\partial r\right)\right]n_r + \left[r\left(\partial A_\varphi/\partial z\right)\right]n_z \\
&= A_1 \sum_{k=0}^{\infty}\Big\{\left[\left(4.(-1)^k\right)/(2k+1)\pi\right]\cos\left[\left((2k+1)(\pi/b)\right)z\right] \\
&\quad .\left[\left(r.\lambda.J_0(r\lambda)\right)/\left(J_1\left[(r_0+a)\lambda\right]\right)\right]\Big\}.n_r \qquad (7) \\
&\quad + A_1 \sum_{k=0}^{\infty} -\Big\{\left[\left(4.(-1)^k\right)/(b)\right]\sin\left[\left((2k+1)(\pi/b)\right)z\right] \\
&\quad .\left[\left(J_1(r\lambda)\right)/\left(J_1\left((r_0+a)\ \lambda\right)\right)\right]\Big\}.n_z
\end{aligned}
$$

Where $J_0(r\lambda)=\sum_{m=0}^{\infty}\left(\left[(-1)^m(\lambda r/2)^{2m}\right]/\left[(m!)^2\right]\right)$ is the Bessel's function of zero order. n_r and n_z are the normal vector n components in the (r,z) plane.

In the finite element discretized area (FEDA), the Galerkin formulation of equation (1) is given by the following relation:

$$
\begin{aligned}
&\iint_\Omega \nu\left[(\partial A/\partial r)(\partial\alpha_i/\partial r)+(\partial A/\partial z)(\partial\alpha_i/\partial z)\right](drdz/r) \\
&+\iint_\Omega \sigma(DA/Dt)\alpha_i(drdz/r)-\int_\Gamma H_t\,\alpha_i d\Gamma \qquad (8) \\
&=-\iint_\Omega \sigma\alpha_i\left[(1/r)(\partial V/\partial\varphi)\right]drdz
\end{aligned}
$$

where α is the projection function, Ω is the FEDA and Γ is the interface between the FEDA and the MZ. So, the integral term considered on Γ can be expressed using the field H_t formulae (6).

When the load moves respect to the inductor, only the MZ formulae (6) has to be changed through the modification of the coordinates r and z. It can be noticed that the MZ formulae can be associated to standard software since it can be put as Newmann boundary condition.

ELECTROMACHANICAL COUPLING MODEL

The dynamical behavior of linear electromagnetic actuator can be basically described by the following electromechanical equations system [6]-[7] :

$$\begin{cases} V(t) = Ri + N\left(\partial\phi(z,i)/\partial t\right) \\ \phi(z,i) = iL(z,i) \\ F_{mag}(z,i) = M\left(d^2z/dt^2\right) + \alpha\left(dz/dt\right) \pm F_g \\ v = dz/dt \end{cases} \quad (9)$$

where $V(t)$ is the exciting voltage applied to the coil, R is the coil resistance, i is the coil current, N is the number of turns, $\phi(z,i)$ is the flux through the coil, z is the displacement, t is the time, $L(z,i)$ is the inductance, $F_{mag}(z,i)$ is the global magnetic force, M is the mobile part mass, α is the friction coefficient, F_g is the force of gravity and v is the mobile part velocity.

The unknown variables of the electromechanical problem are the current (i) and the mechanical displacement (z). The method consists of simultaneously solving the equations system (1); that requires the knowledge of magnetic force (F_{mag}) and flux (ϕ) which are functions of the displacement and the current. These variables (F_{mag}, ϕ) are parameterized using interpolation functions and finite element solution for the electromagnetic equation. This solution is carried out for series of discrete values of the excitation J and the displacement z in the ranges of their real variations. The displacement of the moving part is considered by only the modification of the points coordinates of the interface between both analytical and numerical sub-domains. In this way, the matrix due to the finite element discretization is calculated once and used for every relative position of the moving part.

APPLICATION AND RESULTS

Figure 2 describes the test problem. This is an axisymmetrical actuator used to produce strike forces. It is composed by too coils and a cylindrical steel armature moving following -z- axis when a voltage stem from a capacitor discharge is applied to coil 1 or coil 2.

The characteristics of the system are : $M = 5.52\ kg$, $\nu = 4.35.10^{-3}$ (the relative reluctivity of the armature), $\sigma = 0.715.10^{6}\ S/m$ (armature), $R_1 = 3.21\ \Omega$, $R_2 = 1.22\ \Omega$ and $\lambda = 138.98\ N.s/m$.

Figure 3, 4, 5 and 6 show respectively the supplying voltage, the electrical current in the coil, the mechanical displacement of the armature and the velocity as functions of time. Note that, the coil 1 is excited from t = 0 s to t = 0.03 s, after that, the coil 2 will be excited from t = 0.072 s to t = 0.125 s.

In figure 4, the coupling model is compared to the experimental data. One observes that in this figure, the current in coil 1 reaches it's maximum in advance of 62.4 ms then that of the mechanical displacement (Fig. 5). The electromagnetic system is more rapid then the mechanical one.

Figure 6 shows the velocity as a function of time. One observes that, the moving part (armature) arrived at its initial position with a velocity of $v \approx 2.3 \quad m/s$, that corresponds to kinetics energy of $W \approx 14.6 \quad J$.

On the other hand, the electrical conductivity (σ) of the moving part is weak, which permits to neglect the eddy currents in the conductor and allowing the use of the parameterization method.

The computed results based on coupled analytical and finite element analysis for linear electromagnetic actuator having moving parts are in good agreement with the experimental ones (Fig. 4). The difference between measured and calculated values is essentially due to the approximation of the actuator physical properties.

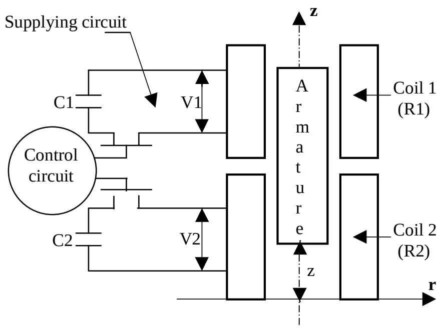

Fig. 2. The axisymmetrical representation of the electromagnetic actuator. Note that : V_1 and V_2 are the supplying voltages. z is the direction of the mechanical displacement of the moving part (armature).

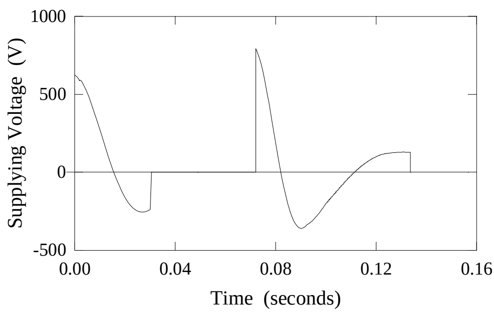

Fig. 3. Supplying voltage as a function of time.

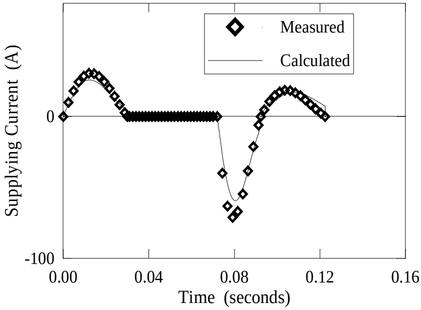

Fig. 4. Supplying current as a function of time.

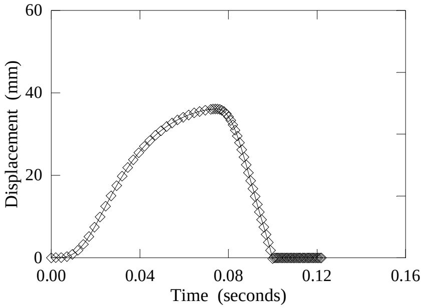

Fig. 5. Mechanical displacement as a function of time.

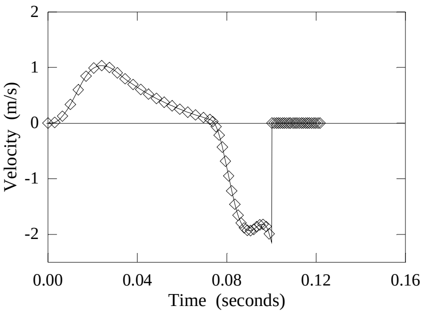

Fig. 6. Velocity as a function of time.

CONCLUSION

The proposed method is elaborated for the modeling of the electromagnetic systems having variable configurations in time. The movement simulation is carried out through the modification of the interface continuity conditions only. Such conditions are obtained from analytical solution, leading to an accurate and economic modeling.

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