Determination of the loads acting on the pump rod of a water pumping windmill

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Abstract - *One of the most delicate parts to size in a water pumping windmill is the pump rod. Indeed, frequent ruptures during the exploitation and experimentation of windmills shows that the determination of the real forces acting on the pump rod is quite difficult to evaluate. The calculation of these forces is traditionally achieved by taking as a maximal force the sum of the static loads. The calculation model used in this article takes into account the hydraulic pressure losses, inertia forces and the shock force caused by the water column on the piston. This will make it possible to better know the forces acting on the pump rod and therefore to perform better sizing.*

Résumé - *Une des parties les plus délicates à dimensionner dans une éolienne de pompage est le train de tige. En effet, comme le montrent les fréquentes ruptures observées au cours de l'exploitation et de l'expérimentation des éoliennes, la détermination des forces réelles s'exerçant sur le train de tige reste assez difficile. Les calculs sont traditionnellement réalisés en prenant comme force maximale la somme des charges statiques. Le modèle développé par le C.W.D (Consulting Wind Energy Developing Countries) est intéressant, car il prend en compte les problèmes de pertes de charges hydrauliques, les forces d'inertie et introduit une force due au choc de la colonne d'eau sur le piston lors de la fermeture du clapet de refoulement. L'utilisation de ce modèle nous a permis d'élaborer un code de calcul qui se rapproche le plus du comportement réel de la tige. Ce code permet de déterminer l'ensemble des forces, ainsi que la force totale agissant sur la tige à différentes positions de la roue. Il permet également de déterminer les vitesses et les accélérations du piston.*

Keywords: Windmill - Pumping - Loads - Force - Pump rod.

1. INTRODUCTION

The calculation of the forces acting on the pump rod of a water pumping windmill used in this report is based on the model developed by C.W.D (Consulting Wind Energy Developing Countries) which is very interesting since it takes into account not only the static forces but also the forces caused by hydraulic pressure losses, inertia forces, and bring in a force caused by the shock of the water column on the piston when the output valve closes.

The use of this model enabled us to work out a computer program which describes closely the real behaviour of the pump rod while the windmill is working and allows the determination of all the forces and in particular the total force at different positions of the rotor. It can also determine the velocities and accelerations of the piston.

The program can be briefly described as follows:

From files or by entering the data of the rotor, transmission and pump this program determines all the forces acting on the pump rod while the water pumping windmill is working.

These forces vary with respect to the rotation angle of the rotor in one hand and the wind speed on the other hand.

2. DESCRIPTION OF THE FORCES ACTING ON THE PUMP ROD

The forces acting on the pump rod are the following :

- static force caused by the water column,
- static force caused by the piston and the pump rod weight,
- friction force caused by the friction of the piston and the cylinder,
- friction force caused by the water flow in the pipe,
- friction force caused by the piston valve,
- inertia force of the piston and the pump rod,
- inertia force of the water column,
- force caused by the shock between the valve and the piston.

These forces are represented in Figs. 1-5.

♦ **Static force caused by the water column : F_{statw}**

When the piston valve is closed and the foot valve is open (delivery), the static force due to the water column is :

$$F_{statw} = \rho_w \cdot g \cdot HMT \cdot \left(A_p - A_{pr}\right)$$

♦ **Static force caused by the piston and pump rod weight : $F_{statppr}$**

This force is constant and does not depend on the crank position. This is given by :

$$F_{statppr} = \left(M_{prm} \cdot L_{pr} + M_p\right) \cdot g$$

♦ **Friction force between the piston and the cylinder : F_{frcup}**

Between the closure of the piston valve and the bottom dead center, the forces acting on the piston cup are F_{statw}, F_{accw}, F_{frw} :

$$F_{frcup} = \frac{\mu \cdot \pi \cdot D_p \cdot H_{cup} \cdot \left(F_{accw} + F_{staw} + F_{frw}\right)}{\left(A_p - A_{pr}\right)}$$

♦ **Friction force caused by the water flow in the pipe : F_{frw}**

This is given by the following expression :

$$F_{frw} = \Delta p \cdot \left(A_p - A_{pr}\right)$$

Head loss Δp : $\Delta p = \Delta p_1 + \Delta p_2$

The head loss at the tee and the piston-pipe connection is : $\Delta p_1 = Z_t \cdot \rho_w \cdot \frac{v_e^2}{2}$

The pressure drop in the pipe is : $\Delta p_2 = \left(\frac{F \cdot HMT}{D_h}\right) \cdot \left(\rho_W \cdot \frac{v_e^2}{2}\right)$

The rate of flow Q in the pipe is given by : $Q = \eta_{vp} \cdot V_p \cdot \left(A_p - A_{pr}\right)$

The pipe flow velocity V_e is : $V_e = \frac{Q}{\left(A_{rm} - A_{pr}\right)}$

The Reynolds number Re is : $$Re = \frac{V_e \, . \, D_h}{\rho_w}$$

Flow friction coefficient F :

For a rough pipe (Nikouradzé formula [4]) :

A pipe wall is said rough when one of the values of K / D_h are higher than 0.001.

$$F = \left(2 \, \log\left(\frac{D_h}{2K} \right) + 1.74 \right)^{-2}$$

For a smooth pipe (Blasius formula [4]) : $$F = \frac{0.316}{Re^{1/4}}$$

Singular head loss coefficient Z_t [3] : $$Z_t = Z_1 + Z_2$$

At the piston-pipe connection Z_1 : $$Z_1 = 0.5 \left(1 - \frac{D_p}{D_h} \right)$$

At the tee connection Z_2 : $$Z_2 = 20 \, . \, FF$$

♦ **Friction force caused by the delivery valve : F_{frpv}**

During suction (at the top dead centre where the valve closes), the friction force at the piston valve is :

$$F_{frpv} = - \left(Z_v \, . \, \rho_w \, . \, \frac{{V_p}^2}{2} \right) . \, A_p$$

Head loss coefficient : $$Z_v = \alpha_0 + \beta_0$$

where $$\alpha_0 = 0.55 + 4\left(\frac{b_t}{D_0} - 0.1 \right) \qquad \beta_0 = \frac{0.155}{\left(\frac{H_{lv}}{D_0} \right)^2}$$

This formula is valid in the following limits :

$$0.1 < \frac{H_{lv}}{D_0} < 0.25 \quad \text{and} \quad 0.1 < \frac{b_t}{D_0} < 0.25$$

♦ **Piston velocity and acceleration**

The velocity and acceleration expressions are as follow :

$$V_p = R_{cranck} \, . \, \omega \, . \left[\sin\alpha + \left(\lambda / 2 \right) . \, \sin 2\alpha \right]$$

$$A_p = R_{cranck} \, . \, \omega^2 \, . \left[\cos\alpha + \lambda . \cos 2\alpha \right]$$

with :

$$\lambda = \frac{R_{crank}}{2 . L_{cr}}$$

- **Valve closure angle : α_{pvc}**

When the piston is going down and at the bottom dead centre, the delivery valve does not close at this point but at a different position corresponding to an angle given by [1]:

$$\alpha_{pvc} = \frac{2}{3} \arccos\left[\frac{R_{cranck} - H_{lv}}{R_{crank}}\right]$$

- **Inertia force of the piston and the pump rod : F_{accppr}**

The inertia force on the piston and pump rod masses is :

$$F_{accppr} = \left(M_{prm} . L_{pr} + M_p\right) . A_{ccp}$$

- **Inertia force of the water column: F_{accw}**

The water mass experiences the effects of the piston acceleration and gives the following force :

$$F_{accw} = \rho_w . HMT . \left(A_p - A_{pr}\right) . A_{ccp}$$

- **Force caused by the shock between the valve and the piston : F_{shock}**

At the closure of the valve which corresponds to the angular position of the transmission $\alpha = \alpha_{pvc}$, there is a shock between the water column and the piston.

This force is defined as follows [2] : $F_{shock} = E . A_{pr} . \delta_{dyn} . L_{pr}$

Dynamic deformation : $\delta_{dyn} = \sqrt{\left(\delta_{stat} . \frac{V_{ppvc}^{2}}{g}\right)}$

Static deformation : $\delta_{stat} = \frac{\left(\rho_w . g . A_p . HMT . L_{pr}\right)}{\left(E . A_{pr}\right)}$

- **Total force : F_{totale}**

The determination of the total force depends on the crank rotation angle.

For : $0 < \alpha < \alpha_{pvc}$

$$F_{totale} = F_{accppr} + F_{statppr}$$

For : $\alpha = \alpha_{pvc}$

$$F_{totale} = F_{accppr} + F_{statppr} + F_{accw} + F_{statw} + F_{frw} + F_{frcup} + F_{shock}$$

For : $\alpha_{pvc} < \alpha < 180°$

$$F_{totale} = F_{accppr} + F_{statppr} + F_{accw} + F_{statw} + F_{frw} + F_{frcup}$$

For : $180° < \alpha < 360°$

$$F_{totale} = F_{accppr} + F_{statppr} + F_{frpv}$$

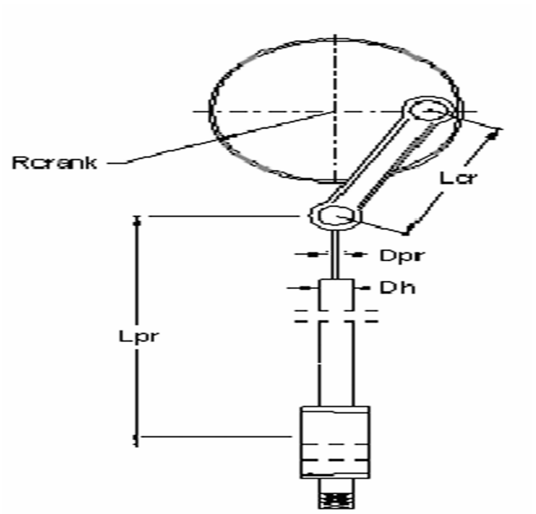

Fig. 1 : Geometrical characteristics of transmission and pump

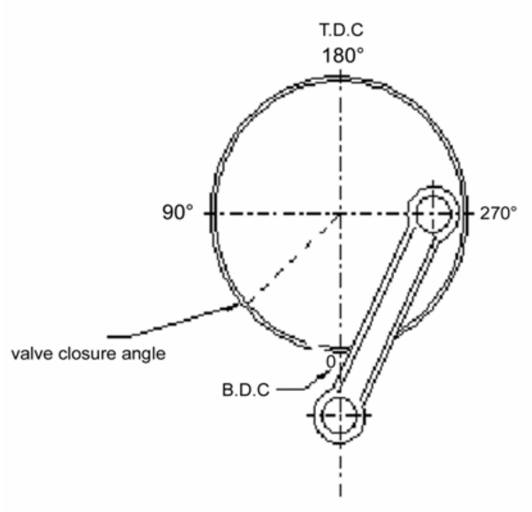

Fig. 2 : Crank angular position

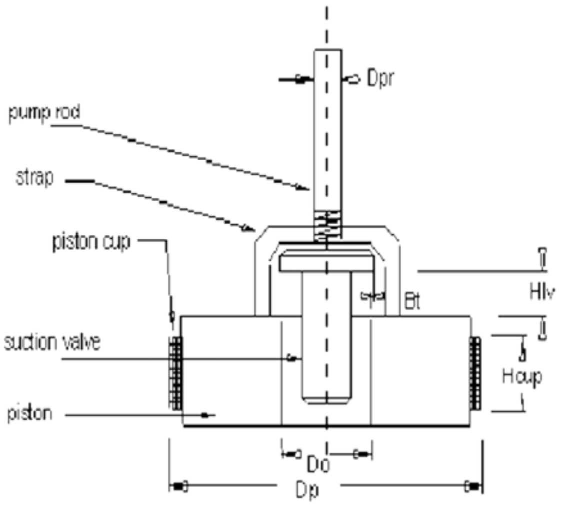

Fig. 3: Characteristics of piston pump

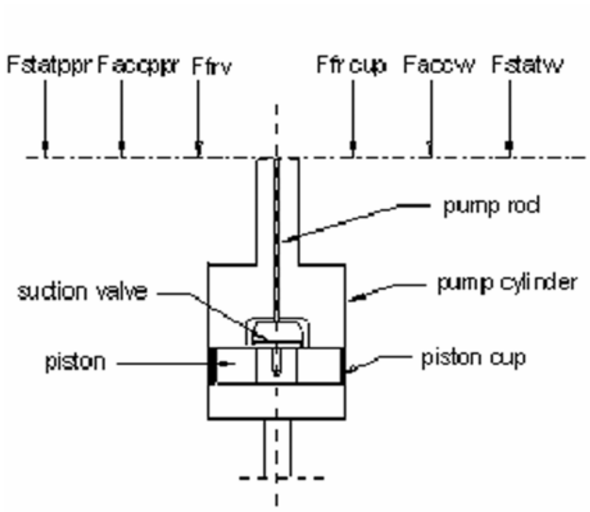

Fig. 4: Forces acting on the pump rod between valve closure and T.D.C

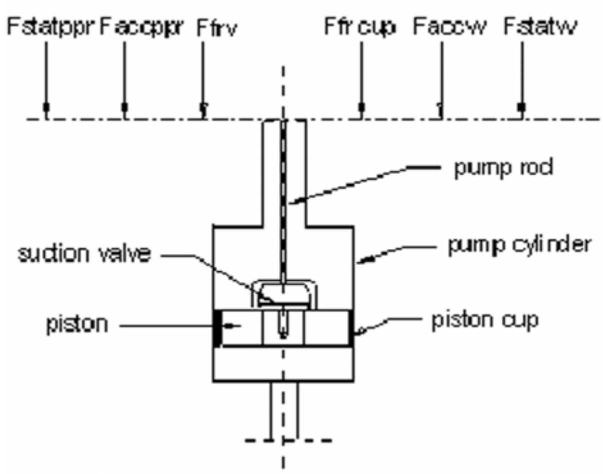

Fig. 5: Forces acting on the pump rod between T.D.C and B.D.C

3. RESULTS OF THE MAIN FORCES ACTING ON THE PUMP ROD OF A WINDMILL

A windmill developed in our laboratory has been taken as an example for the calculation of the main forces acting on the pump rod. The results are shown in table 1 and in Figs. 6-9.

Some characteristics of this windmill are :

Rotor diameter:	2,5 m
Number of blades:	8
Design velocity:	3 m/s
Pumping head:	15 m
Pump diameter:	0.08 m
Stroke:	0.08 m
Rate of flow:	450 l/h

Table 1: Maximum and minimum forces acting on the pump rod of the windmill

Velocity (m/s)	ω (rad/s)	F_{max} (N)	F_{min} (N)
5	8.15	3980	213
6	10.60	5443	193
7	12.95	7009	169
8	15.23	8688	141
9	17.47	10485	109
10	19.67	12403	73

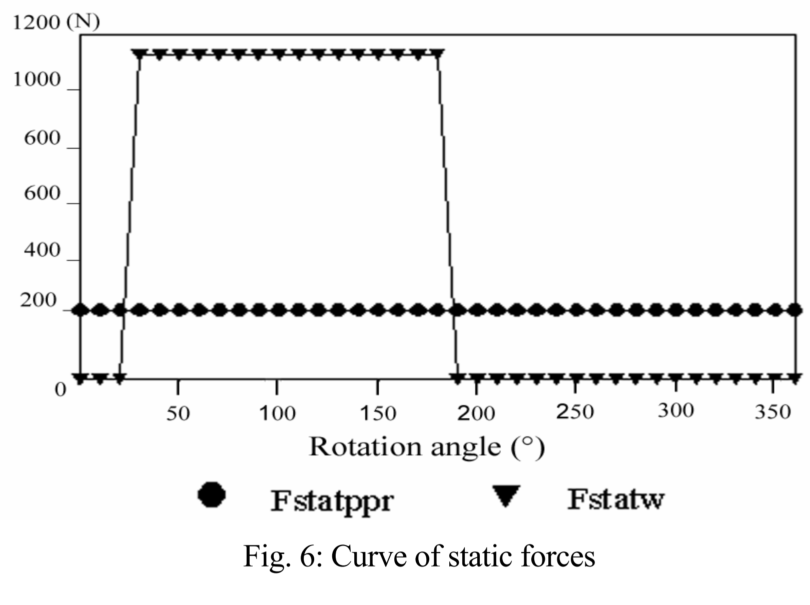

Fig. 6: Curve of static forces

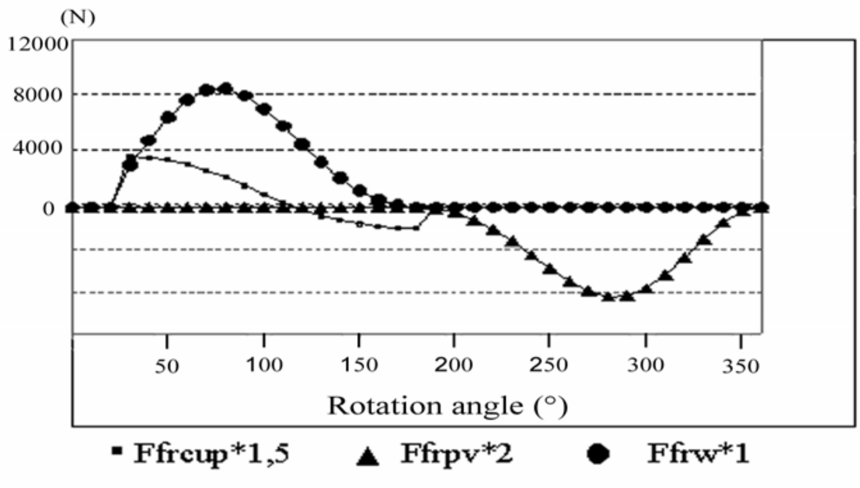

Fig. 7: Curve of friction forces

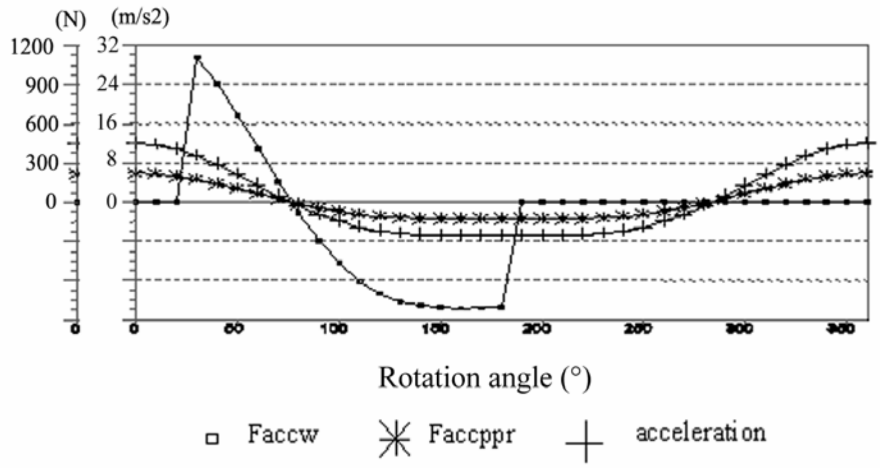

Fig. 8: Curve of acceleration forces

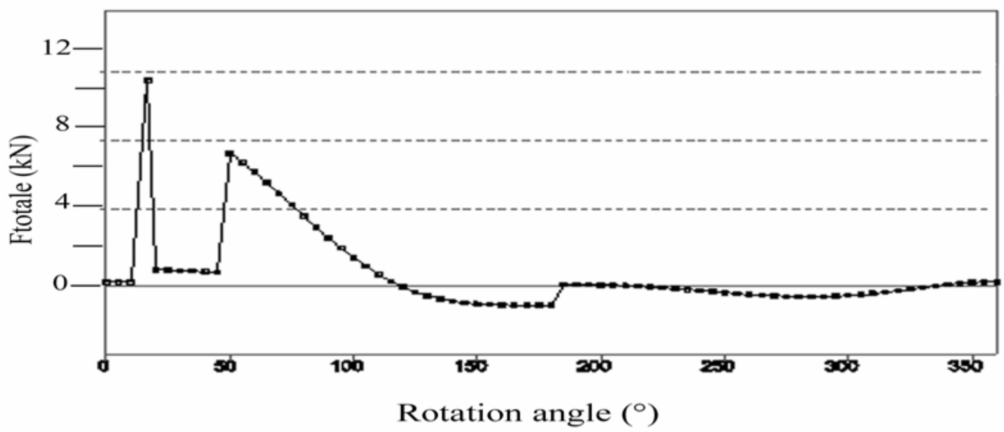

Fig. 9: Curve of the total force on the pump rod

4. CONCLUSION

Some forces of the model developed by C.W.D were modified for an established theoretical approach (see bibliographical references), in particular for the determination of the shock force and the friction force at the valve.

This work provides a better insight on how the forces act while the windmill is operating. Therefore a better sizing will be possible resulting in a more reliable windmill.

The analysis of the results given by this model shows that the maximal force appears at the closure of the valve, i.e. when the piston is going up, it shows also the high variation of the forces acting on the pump rod. This confirms the essential importance in taking account of fatigue when sizing.

NOMENCLATURE

A_{ccp} :	Piston acceleration (m/s^2)	H_{cup} :	Cup height (m)
α :	Crank rotation angle (°)	H_{lv} :	Valve gap height (m)
α_{pvc} :	Valve closure angle (°)	HMT :	Pumping height (m)
A_p :	Piston area (m^2)	K :	Pipe wall roughness height
A_{pr} :	Pump rod cross section area (m^2)	L_{cr} :	Crank length (m)
A_{rm} :	Pipe cross section area (m^2)	L_{pr} :	Pump rod length (m)
b_t :	Valve lapped width (m)	M_p :	Piston mass (kg)
D_0 :	Piston water outlet diameter (m)	M_{prm} :	Pump rod linear mass (kg/m)
D_h :	Pipe diameter (m)	μ :	Cup friction coefficient
D_p :	Piston diameter (m)	ω :	Rotor angular velocity (rad/s)
E :	Modulus of elasticity (Young) (N/m^2)	Q :	Instantaneous flow rate (m^3/s)
η_{vp}:	Pump volumetric efficiency	Δ_p :	Pressure loss (N/m^2)
F :	Pipe friction coefficient	R_{cranck} :	Crank radius (m)
FF :	Friction factor	Re :	Reynolds number
F_{accppr} :	Piston and pump rod inertia force (N)	ρ_w :	Water density (kg/m^3)
F_{accw} :	Water column inertia force (N)	V_e :	Water flow velocity (m/s)
F_{frcup} :	Friction force between the piston and the cylinder (N)	V_p :	Piston velocity (m/s)
F_{frpv} :	Friction force due to the delivery valve (N)	Z_1 :	Piston-pipe head loss coefficient
F_{frw} :	Friction force due to the flow in the pipe (N)	Z_2 :	Tee connection local head loss coefficient
F_{shock} :	Force due to the shock between the valve and the piston (N)	Z_t :	Total head loss coefficient
$F_{statppr}$:	Static force due to the piston and pump rod weight (N)	Z_v :	Valve head loss coefficient
F_{statw} :	Static force due to the water column (N)	g :	Acceleration due to gravity (m/s^2)
F_{totale} :	Total force (N)		

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