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Electromagnetic characterization and modelling of superconducting material

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ABSTRACT

Electromagnetic characterization and modelling of superconducting material are very important for end users. Firstly main features of superconducting material are presented. Secondly two methods of characterization to obtain experimentally the influence of magnetic field on critical current density JC are presented. These two methods are commonly used for applications in electrical engineering. The electrical method is based on measuring the voltage and current of a superconducting sample. It has the advantage of simplicity in implementation, but the main drawback as the presence of the self magnetic field. This prevents the determination of the parameters of $J_C(B)$ for weak magnetic fields. The magnetic method using the cycle of magnetization has the advantage of making measurements without contact with the sample but has two drawbacks : it is based on a theoretical model in which J_C is constant and the assumption of infinite length sample.

1. Introduction

The superconducting materials are characterized by three critical quantities that are critical temperature T_C, the critical magnetic field H_C and the critical current density J_C (Wilson, 1983) (Tixador, 1995). The material is superconducting below a critical surface (Figure 1) beyond the material is in normal state. In applications using these superconducting materials, such creation of high magnetic field, magnetic levitation, current transport, motors and transformers, the knowledge of the critical current density J_C of the material is paramount. Thus in the magnetic field coils J_C is related to the maximal magnetic field. In levitation systems J_C sets the power of levitation. In power cable J_C determine AC losses. Finally in electric motors, torque is directly related to J_C. The critical current density is dependent on two other parameters, magnetic field B and temperature T. So the law $J_C(B, T)$ has to be determine. In this article, only $J_C(B)$ determination is exposed. Temperature T is considered constant in superconducting samples.

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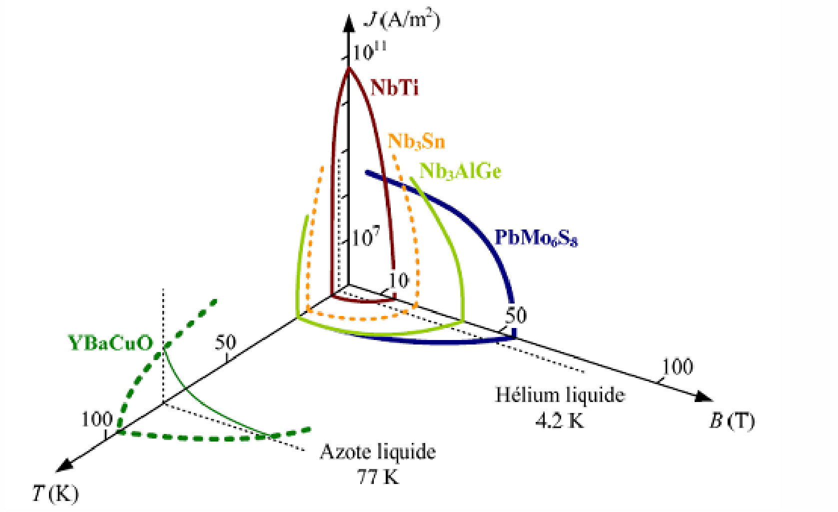

Fig. 1. Critical surface of different superconducting materials

2. Critical magnetic fields

Beyond a critical magnetic field value, the superconductivity disappears. In this regard, two different behaviours define two types of superconductors.

Superconductors of type 1 (Figure 2.a) are characterized by a single critical field H_C. When subjected to an external magnetic field H the magnetic induction B is zero for $H < HC$ and therefore magnetization $M = -H$. Because of their low critical magnetic field, these materials are without interest in electrical engineering.

Type 2 superconductors have two critical field H_{C1} et H_{C2} (Figure 2.b) with $H_{C2} \gg H_{C1}$. They are considered as superconducting state if H is less than H_{C2}. Magnetic induction $\mu_0 \cdot H_{C2}$ is greater than or much greater than the Tesla, so they are more suitable for practical use.

The evolution of the macroscopic magnetic induction B as a function of applied magnetic field H differs from type I between H_{C1} and H_{C2} (Figure 1.2). In this area type II superconductors no longer possesses the property of perfect diamagnetism, screening is partial. It says in a mixed state.

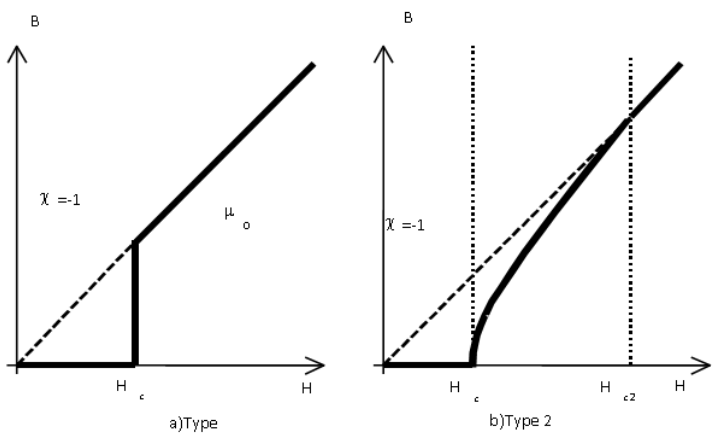

Fig. 2. PV generator monitoring using artificial neural network.

In the mixed state at the microscopic level, there is penetration of magnetic induction in the material as tubes or vortex. They are arranged in triangular lattice Abrikosov or network to minimize the energy of this network. The distance d between the vortices varies with the magnetic induction B :

$$d = k\left[\left(\frac{\varphi_0}{B}\right)^{\frac{1}{2}}\right]$$

Each vortex (Figure 3) has the same quantum of magnetic flux $\varphi_0 = 2,07.10^{-15}Wb$. Its radius is equal to the coherence length ξ. Around these tubes are formed of superconducting shielding currents and the microscopic magnetic induction $B\mu$ decreases exponentially in thickness λ_L. In type II superconductors the coherence length is less than the length of London.

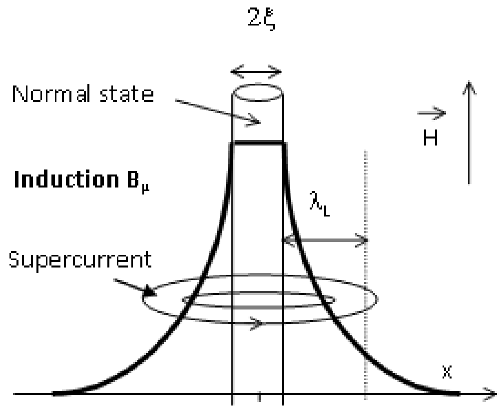

Fig. 3. Vortex structure.

In Figure 4 is represented the case of a superconducting cylinder of type 2 subject to axial magnetic field. Between H_{C1} and H_{C2} the number of vortices increases gradually and eventually occupies the entire material. The magnetic induction B in the material passes so gradually $\mu_0 H_{C1}$ at $\mu_0 H$ (Figure 2).

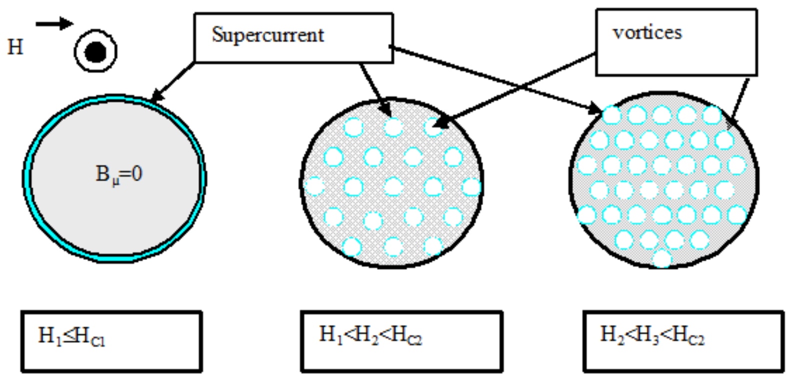

Fig. 4. Increase of B in a type 2 superconducting cylinder between H_{C1} and H_{C2}.

3. E(J) law of superconductor

To remain superconducting state, the current density J in the material must remain below a critical value J_C. This value depends on B and temperature T. The critical current density J_C is linked to the phenomenon of vortex pinning.

One of the problems of applied superconductivity notation is the definition of different current density. These may be present in the material or measured experimentally. In the first case definitions most commonly used are the following :

J_S is the current density of the supercurrent at the microscopic level that is circulating around the vortex and that is circulating at the surface of the material when $H < H_{C1}$. Its value is about $10^7 A/cm^2$.

J_C is the macroscopic critical current density is related to the local magnetic field H by the macroscopic Maxwell's equation follows :

$$\overrightarrow{rot}\overrightarrow{H} = \overrightarrow{J} = \overrightarrow{J}_C \tag{1}$$

In case of high critical temperature superconductors, materials that are anisotropic, J_C is in several different values for different directions.

Values of critical currents are obtained experimentally or by an electrical method (J_{CT}) or by a magnetic method (J_{CM}). In the first case transport current traverses the sample. In the second case superconducting sample is subjected to a magnetic field.

The difficulty is obviously to move from the measured quantity to the real quantity in the material.

Superconductors of type 2, in which there are vortices, cannot carry current without dissipating. If there is a current in a superconductor with a direction perpendicular to the external magnetic field which creates the vortex, it produces two phenomena (figure 5).

First the self magnetic field created by the transport current creates an imbalance between an area of strong magnetic field on one side of the superconductor and a low field region to the other side.

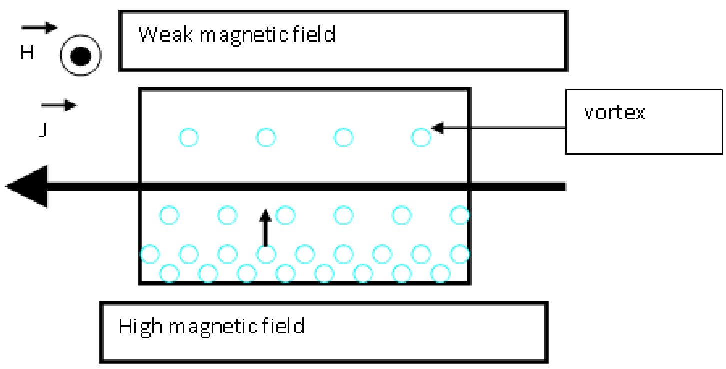

Fig. 5. Moving vortex in a superconducting material traversed by an transport current perpendicular to the direction of the vortex.

On the other hand, the vortices are subjected to a Lorentz force if the current direction is different from the applied external field direction :

$$\overrightarrow{F}_L = \overrightarrow{J} \wedge \overrightarrow{B} \tag{2}$$

The vortices will move from a region of strong magnetic field to a region of low field where they disappear, generating an electric field and therefore energy dissipation. This results in a resistivity ρ_f dependent of B and resistivity in the normal state ρ_n :

$$\rho_f = \left(\frac{B}{B_{C2}}\right)\rho_n \quad (3)$$

The curve $B(H)$ of type 2 superconductors (Figure 2) is actually a curve that exists only in the case of a perfect crystal of superconductor, without defects and impurities. If you submit this material to a magnetic field H the vortices are distributed uniformly in the section perpendicular to H as shown in Figure 4. This therefore produces a macroscopic magnetic induction B uniform in this section. This is not the case most of the time in the materials used in reality. In these materials the defects (dislocations, impurities, grain boundaries etc.) will block the vortex at pinning sites. In this case if the material is subjected to a magnetic field (Figure 4), the distribution of vortices in the cylinder is different from the previous case (Figure 6). If H is less than or equal to H_{C1} there is no difference because there is not vortex. If H is greater than H_{C1} and lower than H_{C2} vortices first appear on the outer surface or they are trapped. This results in the presence of two distinct zones. A first outer zone with presence of vortices where B is different from zero and an inner zone without vortex where B is zero. This spatial variation of B creates current densities induced in the superconducting material. C.P. Bean has shown that these induced currents have a current density equal to the critical current density J_C. That is the critical state model of Bean.

Between H_{C1} and H_{C2} is defined the full penetration magnetic field H_P for which the vortices and the current density fully penetrate the material. Beyond H_P the number of vortices and therefore B increases to B_{C2}. The current density still exists in this case throughout the material but decreases when B increases until zero at $H = H_{C2}$.

To obtain high J_C materials is important to have the largest number of defects. This property of materials with trapping sites contain large volume flows, has led to the development of applications such as magnetic field shielding and trapping of magnetic flux for the realization of superconducting magnets.

In practical, type 2 superconductors are used only with pinning sites and therefore we refer only to them later in this article.

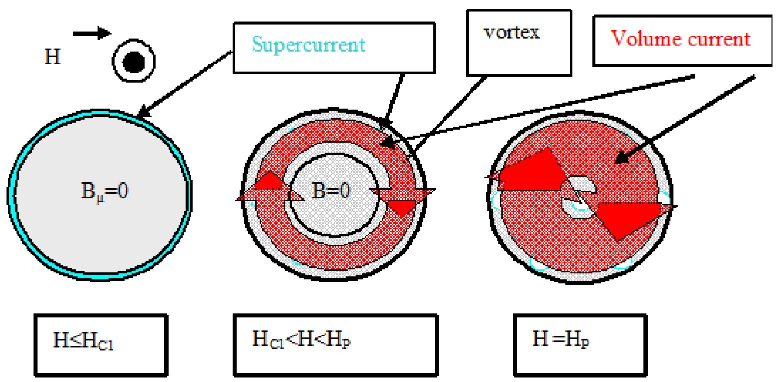

Fig. 6. Increase of B for H between H_{C1} and H_{C2} in a type 2 superconducting cylinder with pinning.

In a non-superconducting conductive material relationship between the electric field E and current density is $E = \rho \cdot J$. The characteristic $E(J)$ of a superconductor is very different and varies depending on temperature and magnetic field. In this section we consider a superconducting material in which these vortices are created by an external field and a current transmission direction perpendicular to the direction of the vortices. These vortices are therefore subject to the Lorentz force defined above.

In Figure 7 the characteristic $E(J)$ of a superconductor at very low temperatures is shown. For small values of J less than J_C, the vortices are embedded as the Lorentz force is less than the pinning force and in this case $E = 0$. The critical current density is one for which the Lorentz force F_L is greater than the pinning force F_P causing a shift to the state of "flux flow". In this case the vortices move together and the losses are equivalent to those due to viscous friction. The critical current density is one that will untie the vortex from defects causing a transition to the state of "flux flow". The growth of E is very large when $J > J_C$ and thus the material passes very quickly to the normal state when J increases. We can therefore consider that if the material is superconducting and $J = J_C$, this is the critical state model of Bean.

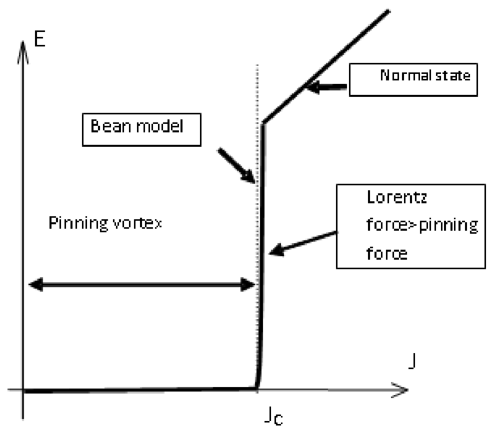

Fig. 7. $E(J)$ law of superconductor at very low temperature.

In Figure 8 the characteristic $E(J)$ of a superconductor at temperatures where the influence of thermal effects is important is shown. The thermal motion makes it likely that the movement of vortices even if $F_L < F_A$ and helps to round off the characteristic $E(J)$ (Figure 8). For low values of E with J close to the J_C curve $E(J)$ is of exponential form, is the region of “flux-creep”. This corresponds to displacement of the vortices due to thermal activation and thus the appearance of a weak electric field but nonzero. For high values of E with $J > J_C$ is the region of flux flow. In practice the operating temperature in applications that use superconducting materials is usually high enough that it can be seen in the latter case. It is obvious that in the case of high critical temperature superconductors used relatively high temperatures these phenomena related to the thermal motion become dominant. The critical current density is the value for which the material passes from the state of “flux creep” in the state of “flux flow”. The transition between these two states is gradual and therefore difficult to identify experimentally. To define J_C in this case we use a critical electric field criterion ($0.1\mu V/cm$ or $1\mu V/cm$ for example) that do not necessarily correspond to the physical border between two states but is a convenient practical criterion for comparing performance of different superconducting materials.

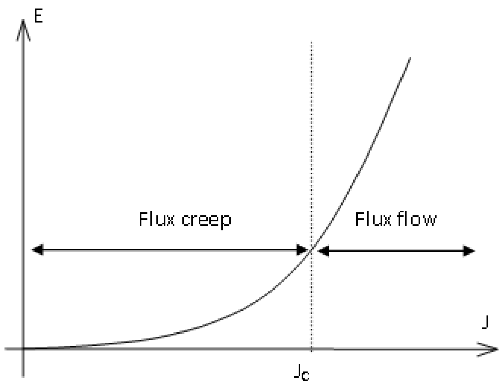

Fig. 8. $E(J)$ law of superconductor at "hot" temperature.

4. $J_C(B)$ laws of superconductor

In superconductor the relation between J and J is :

$$\vec{E} = E_C.\left(\frac{J}{J_C(B,T)}\right)^{n(B,T)}.\frac{\vec{J}}{J} \qquad (4)$$

For LTS, n is large (several tens). But for HTS n varies between 7 and 30. The value of n is determined from experimental curves. If n tends to infinity the relation $E(J)$ is equal to the critical state model developed by Bean (Bean, 1964). This requires that the local density of current in a superconducting material is either zero or equal to its critical value J_C.

$$J = \pm J_C \ ou \ J = 0 \qquad (5)$$

In [4], the critical current density is defined for a criterion of critical electric field $EC(0.1, 1$ or $10\mu V/cm)$. For HTS, n is relatively small and the critical current varies enormously depending on the test. E_C value most used for HTS is $1\mu V/cm$.

The most commonly used $J_C(B)$ relationships in literature are (Table 1) models of Bean (Bean, 1964), Kim (Kim, 1962) and linear (Watson, 1968) (Douine et al., 2006) . The Bean model is widely used in theoretical calculations because it allows simple analytical calculations of current distribution in a superconducting material such as a plate or cylinder. The linear model is valid especially for LTS and allows analytic calculations of losses (Douine et al., 2008). The model of Kim is valid for granular first-generation HTS, where the flow of transport current is a intergranular current.

Table 1. $J_C(B)$ law.

$J_{CB} = J_{c0}$ independent of B	Bean (6)
$J_{CK}(B) = \frac{J_{C0}}{\left(1+\frac{\lvert B\rvert}{B_{K0}}\right)}$	Kim (7)
$J_{CL}(B) = \frac{J_{C0}(B_{j0}-\lvert B\rvert)}{B_{j0}}$	Linear (8)

J_{C0}, B_{K0} and B_{j0} are positive constants.

5. DC characterization of superconductors

$J_C(B)$ law of a superconducting material can be obtained in several ways (Vanderbemden, 1999) (Senoussi, 1992). This paper will be presented an electrical method, where the sample is in contact with the measuring system it is called direct method, and a magnetic method, where the sample is not in contact with the system of measurement, referred to as indirect method.

5.1. $J_C(B)$ determination electric method

Electrical measurements of critical current density of superconducting materials are usually performed by the 4-point method (Douine et al., 2006) (Senoussi, 1992). The sample is fed by a current I and a voltage U across the sample is measured by two wires (Figure 9). So simple even simplistic E is deduced from U by $U = EL$ with L the distance between the voltage taps and J is deduced from I by $J = J/S$ with S section of the sample. These relationships are theoretically valid only if the current density has fully penetrated the sample and so the current is large enough. However these relationship are generally used also low current. In addition for high currents, thermal effects due to losses in the superconducting material and contact resistances above-copper prevented making perfectly isotherm measurements. J_C is deduced from $E(J)$ with an electric field criterion equal $1\mu V/cm$. To obtain experimental $J_C(B)$ curve (Figure 10), the sample supplied with a current I is subjected to an external magnetic field B_{ext} parallel to its axis. The voltage U is measured as a function of I for different external magnetic fields (Figure 11). To differentiate the local $J_C(B)$ in material and $J_C(B)$ deduced from measurements, this last one is noted $J_{mc}(B_{ext})$ thereafter.

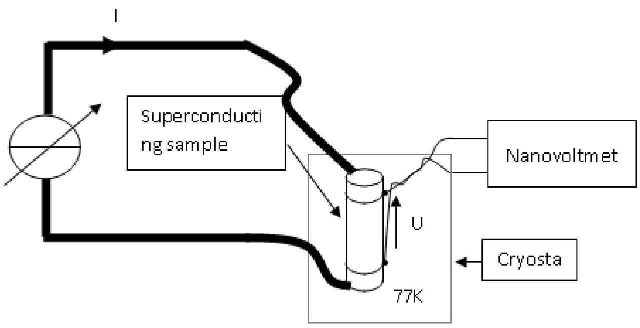

Fig. 9. 4-points electric method.

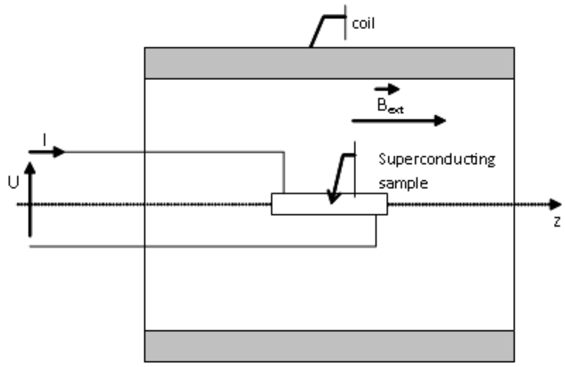

Fig. 10. Experimental bench for $J_C(B)$ electric determination.

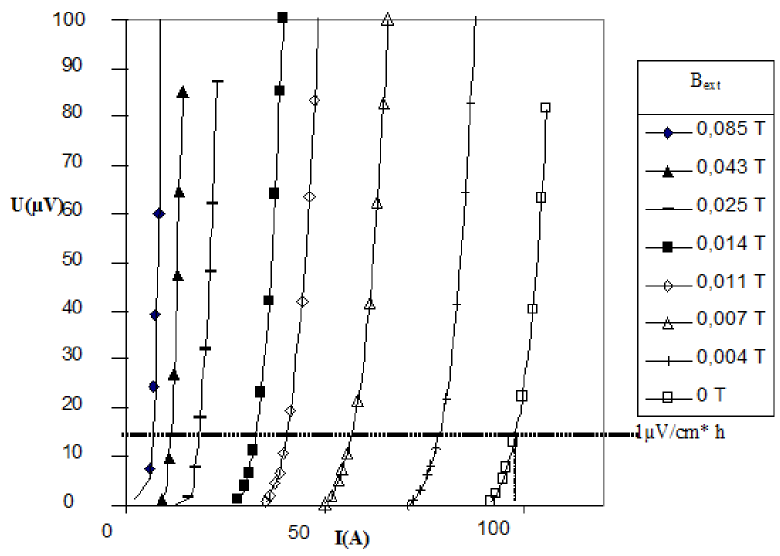

Fig. 11. $U(I)$ curves for different B_{ext}.

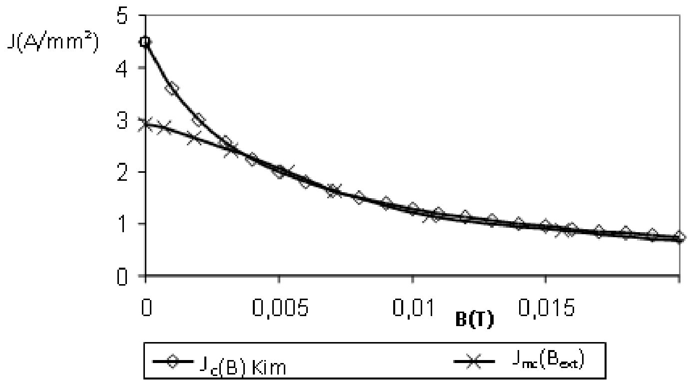

Fig. 12. Experimental $J_{mc}(B_{ext})$ curve and Kim model curve.

Measurements are made (figure 11 and 12) for superconducting tube (internal radius $Ri = 3.8mm$, external radius $Re = 5mm$, section $S = 33mm^2$, distance between electric taps $h = 11.7cm$). For no applied magnetic field, measured critical current $I_{mc0} = 96A$, ($1\mu V/cm$ criteria).

The electrical method is relatively simple to implement but requires a good knowledge of physical phenomena (thermal problems, self-field) involved in the material during measurement.

5.2. $J_C(B)$ determination magnetic method

The electrical method or direct method requires connection of the superconducting sample to a current source. This introduces problems because of the thermal energy input important at low temperature. To avoid this problem methods without electrical connections called indirect methods or magnetic methods exist. They are of two types, DC and AC method. The latter is not described in this article but the reader may find it in a number of references (Vanderbemden, 1999) (Senoussi, 1992). The principle of the

DC method is as follows. The superconducting sample is subjected to an applied magnetic field H_a which causes the onset of screening currents induced in the sample (Figure 13). These induced currents create a magnetic moment m which is likened to a magnetization ($M = m/V$; V is the volume of superconducting material). The local magnetic field H is the vector sum of the applied magnetic field and induced field H_i created by the supercurrent :

$$\overrightarrow{H} = \overrightarrow{H_a} + \overrightarrow{H_i} \tag{9}$$

Average magnetic induction $< B >$ is the sum of applied magnetic field and M :

$$< B >= \mu_0 H_a + \mu_0 M \tag{10}$$

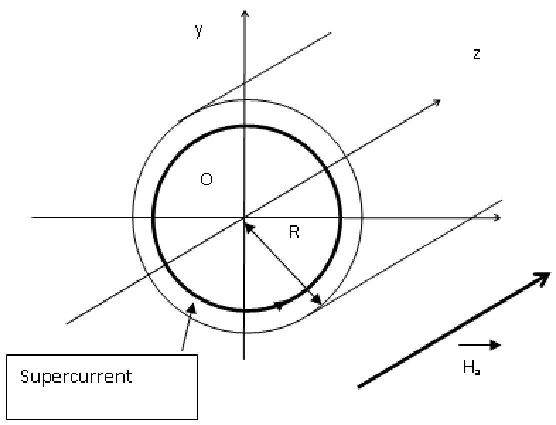

Fig. 13. Superconducting cylinder in applied magnetic field.

To obtain M experimentally the method is as follows. The superconducting sample is placed at the end of a rod connected to a linear motor and in a region subjected to an applied field H_a almost uniform (Figure 14). The measurement is done in two phases (Figure 15). First phase a magnetic field is applied gradually. Second phase, the magnetic field is held constant and the sample is moved upwards. An induced voltage $e(t)$ due to the supercurrent induced in the material is recorded by the measuring system. The curve $M(H_a)$ is deduced (Figure 16) using the following formula

$$M = \frac{< B >}{\mu_0} - H_a = -\frac{\phi_{\max}}{S.\mu_0} - H_a = \frac{\int e(t).dt}{S.\mu_0} - H_a \tag{11}$$

The curve $M(H_a)$ consists of two parts. First, an initial magnetization curve $M_1(Ha)$. It begins as the superconducting material contains no current and ends for $H_a = H_{max}$. Second, a hysteresis loop for H_a from H_{max} to $-H_{max}$.

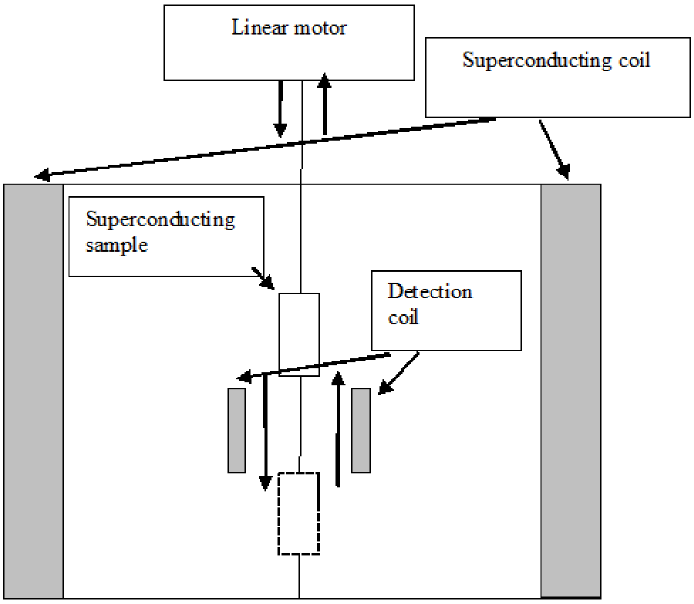

Fig. 14. $M(H_a)$ experimental bench.

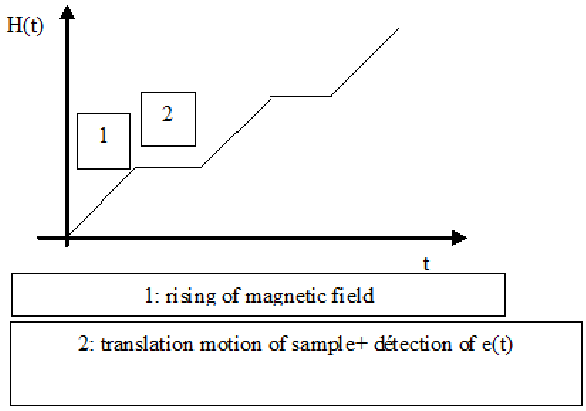

Fig. 15. $M(H_a)$ determination method.

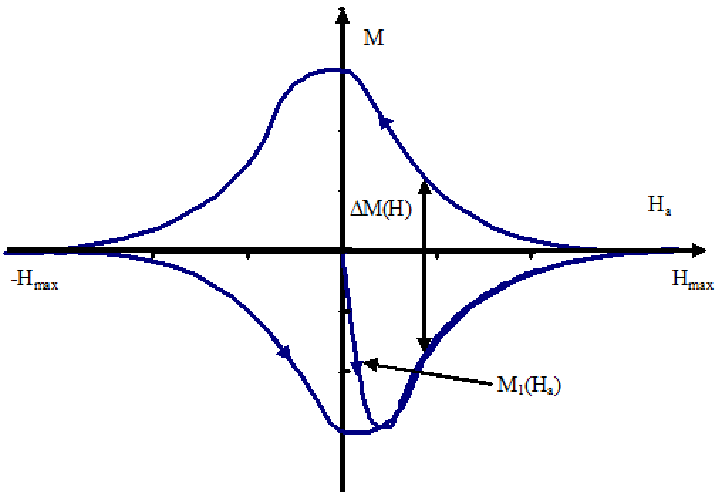

Fig. 16. Measured $M(H_a)$ of superconducting sample.

Currently the most widely used model to deduce $J_C(B)$ from $M(H_a)$ is the critical state model. Bean model (Table 1) is a $J_C(B)$ relationship where J_C is constant. The model of the critical state requires a simple relationship between J_C and H

$$\overrightarrow{rot}\overrightarrow{H} = \overrightarrow{J}_C \tag{12}$$

In the case of a superconducting sample subjected to an axial field the relationship between B and J_C is then (Tixador, 1995) :

$$\frac{dB}{dr} = \mu_0 J_C \tag{13}$$

The distributions of $B(r)$ and $J(r)$ (Figure 17) are deduced from [13]. During the first increase of the applied magnetic field, there is firstly, incomplete penetration of the magnetic field in the cylinder and secondly from the full penetration magnetic field H_{PB} magnetic field is present everywhere in the material and also$J = J_C$ everywhere. H_{PB} is deduced using [13] :

$$H_{PB} = R.J_C \tag{14}$$

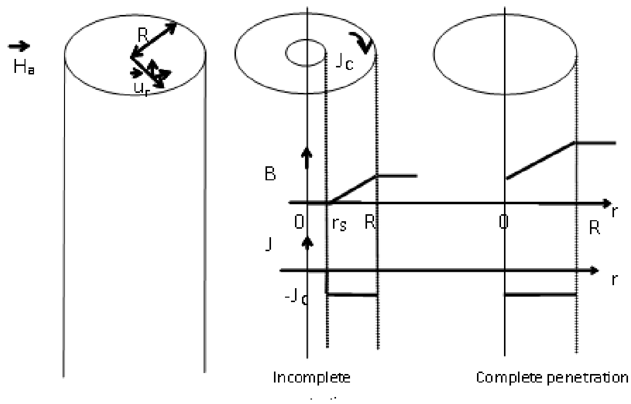

Fig. 17. $B(r)$ and $J(r)$ distribution during first magnetization of superconducting cylinder.

Equation [13] allows calculating the theoretical curve $M(H_a)$ (Figure 1.21). As the experimental curve, it consists of an initial magnetization curve $M_{1B}(H_a)$ and a hysteresis loop. The theoretical curve of first magnetization is also decomposed into two parts. Below H_{PB} there is incomplete penetration of the magnetic field in the sample (subscript I), above H_{PB} there is complete penetration (subscript C) :

$$M_{1BI}(H_a) = -\frac{H_a\left((\mu_0 H_a)^2 - \mu_0^2 H_a.J_{C0}R + 3(\mu_0 J_{C0}R)^2\right)}{3(\mu_0 J_{C0}R)^2} \tag{15}$$

$$M_{1BC}(H_a) = -\frac{J_{C0}R}{3} = -\frac{H_{PB}}{3} \tag{16}$$

There is a simple relationship between the cycle depth ΔM and J_C :

$$J_C = J_{C0} = \frac{3\Delta M}{2R} \tag{17}$$

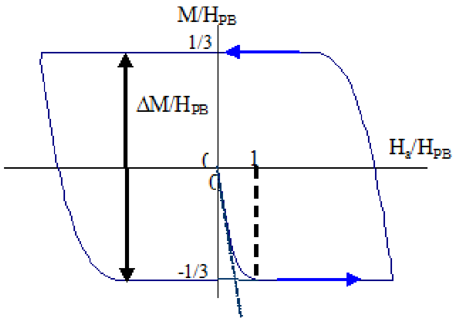

Fig. 18. Bean model theoretical $M(H_a)$ cycle.

The traditional method of determination of $J_C(B)$ (Vanderbemden, 1999) (Senoussi, 1992) (Chen et al., 1989) using the cycle $M(H_a)$ is therefore used [17], calculated using the J_C Bean model which is constant in the extrapolation to cases where J_C is dependant of B :

$$J_C(B) = \frac{3\Delta M(H)}{2R} \tag{18}$$

In Figure 19 is represented the $J_C(B)$ curve of the material which the curve $M(H_a)$ is shown in Figure 16. This is a NbTi cylinder, $2cm$ long and $5mm$ diameter at a temperature of $5°K$. On the same figure is represented the law $J_C(B)$ deduced from Kim model and that is closest to the experimental measurement (Douine et al., 2010).

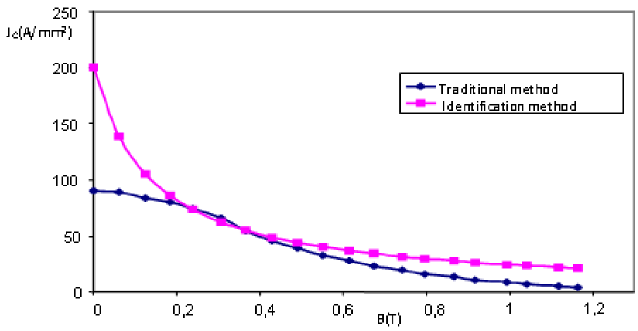

Fig. 19. $J_C(B)$ curves deduced from $M(H)$ measurement and calculated with Bean model and with identification method.

6. Conclusion

Knowledge of the law $J_C(B)$ of a superconducting material is very important for users. In this article two methods of characterization to obtain experimentally the law $J_C(B)$

are presented. These two methods are commonly used for applications in electrical engineering. The electrical method is based on measuring the voltage and current of a superconducting sample. It has the advantage of simplicity in implementation, but the main drawback as the presence of the magnetic field clean. This prevents the determination of the parameters of $J_C(B)$ for weak magnetic fields. The magnetic method using the cycle of magnetization has the advantage of making measurements without contact with the sample but has two drawbacks : it is based on a theoretical model in which J_C is constant and the assumption of infinite length sample.

REFERENCES

[1] Bean C. P., « Magnetization of high field superconductors », Review of modern physics, 1964, p. 31-39.

[2] Douine B., Berger K., Leveque J., Netter D. Rezzoug A., «Influence of Jc(B) on the full penetration current of superconducting tube» Physica C, vol. 443, pp. 23-28, 2006.

[3] Douine B., Berger K., Pienkos J., Leveque J., Netter D., Analytical calculation of the instantaneous power in a current carrying superconducting tube with Jc(B), I.E.E.E. transactions on applied superconductivity, 18(3), pp.1717-1723, september 2008.

[4] Douine B., Leveque J., Mezani S., JC(B) determination method with the help of the virgin magnetization curve of a superconducting cylinder, I.E.E.E. transactions on applied superconductivity, vol. 20(2), pp.82-86,.2010.

[5] Kim Y. B., Hempstead C. F., Strnad A. R., «Critical persistent currents in hard superconductors», Phys. Rev. Lett., volume n°9, 1962, pp. 306-309.

[6] Senoussi S., «Review of the critical current densities and magnetic irreversibilities in high Tc superconductors», Journal de physique III, Vol. 2, 1992, pp. 1041-1257.

[7] Tixador P., Les supraconducteurs, Paris, Hermès, 1995.

[8] Vanderbemden P., «Determination of critical current in bulk high temperature superconductors by magnetic flux profile measuring methods», Ph. D. Thesis, U. Lg. (1999), 193 p.

[9] Watson J. H. P., «Magnetization of Synthetic Filamentary Superconductors, the dependence of the critical current density on temperature and magnetic field», J. Appl. Phys., vol.39, n°7, juin 1968, pp.3406-3413.

[10] Wilson M. N., Superconducting Magnets, Oxford science publications, 1983.