

Reflexion/ Transmission of a plane wave on a plane interface

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Abstract

On a plane interface between two elastic half-space, P and SV waves propagating in the (x, z) plane related by Snell's law and the law of continuity of displacement components u_x and u_z and constraints σ_{zz} and σ_{zx} on both sides of the interface. An incident wave P or wave SV generates two P or SV reflected waves and two transmitted waves P or SV . The four continuity equations are written in the form of a matrix multiplied by a vector transmission-reflection coefficient, defined for potential movement of the particles. For an planar boundary between fluids with different characteristic impedances, there is continuity of u_z and σ_{zz} on both sides of the interface and the shear σ_{zx} in the medium must vanish at the interface (fluid media involving only perfect no viscosity, so that was normal stresses, not shear stress $\sigma_{zx} = 0$). As soon as the angle of incidence exceeds a critical value of incidence, the wave for which the value of incidence is greater than 30° becomes evanescent. The reflection-transmission coefficients become complex.

Keywords : P waves, incidence, reflection, transmission.

1. Introduction

The non destructive characterization of structures has grown considerably in recent years. The ultrasonic methods have become the preferred tool for non destructive evaluation of mechanical properties of materials [1]. They also have the advantage of being applicable to a wide range of materials. Surface waves were a long time the subject of extensive studies which had applications in both non-destructive tests in signal processing [1, 2]. Much research has been conducted on the interaction of such waves with surface discontinuities but most on the reflection and transmission. The elastic waves that result from moving particles propagate only in material media, so that electromagnetic waves propagate in a vacuum also. It was possible to address immediately the propagation of elastic waves in a fluid because this medium is a set of free particles; their properties are expressed using scalar parameters: density ρ , coefficient of compressibility χ , mean free path (average distance traveled by a particle between two collisions). The propagating waves are fully described by a scalar, pressure, or potential expansion of the displacement or velocity [1, 2, 3].

In summary, in a perfect fluid :

- The polarization of the wave, that is to say, the particle motion is necessarily longitudinal, parallel to its wave vector, the absence of viscosity preventing any shearing motion;

- The speed of propagation is expressed by $c = 1/\sqrt{\rho\chi}$;

- The Poynting vector indicating the direction of energy propagation is parallel to the wave vector;

- The polarization of reflected and transmitted waves, on both sides of a surface separate two media of different impedances, and that of the incident wave. Their amplitudes and propagation directions are given by the Snell-Descartes in which only are involved the impedances of the media and the angle of the incident wave.

- The wave continues to propagate when the distance between a maximum and minimum pressure becomes the order of magnitude of the mean free path of particles.

2. Reflection / transmission at a plane interface

Consider the interface between two homogeneous fluids of different velocities (c_1 and c_2); when changing propagation environment, changing the characteristics of a plane wave is particularly interesting. The change in speed causes a specular reflection of the wave in the first medium (in a direction symmetrical to the normal at the point of incidence) and a refraction of the wave in the second medium at an angle given by famous law of Snell.

A progressive plane wave acoustic pressure \hat{p}_a which is of the form $\hat{f}_1(t - \vec{n}_a \cdot \vec{r}/c_1)$, maintained by a source located at infinity (for z tends to $-\infty$), propagates in a half-space fluid (density ρ_1 and speed c_1), bounded by a planar interface located at $z = 0$, separating it from another half-space ρ_2 velocity of c_2) (Figure 1).

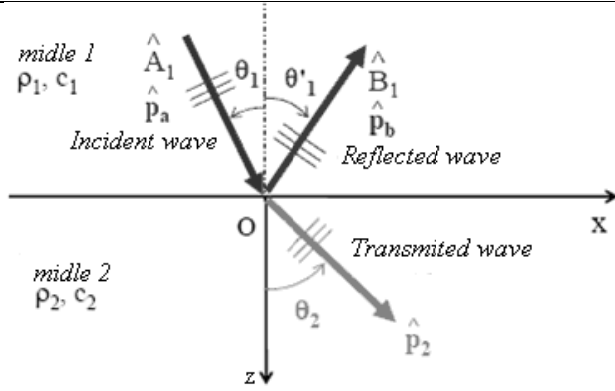


Figure 1. Reflection / transmission at the interface between fluids with different characteristic impedances.

The axes are chosen so that the direction of propagation of the incident wave is located in the plane (Oxz) , and it makes an angle θ_1 with the axis (Oz) (obliquely incident). The interaction of the oblique incident plane wave with the interface generates a reflected wave in the middle F1 amplitude \hat{B}_1 (and direction of propagation making an angle θ'_1 with the axis (Oz)), and a wave transmitted in the middle F2 amplitude \hat{A}_2 (and direction of propagation making an angle θ'_2 with the axis (Oz)).

2. 1. Writing the problem in the middle F1

The propagation equation in the fluid medium is written F1

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c_1^2} \frac{\partial^2}{\partial t^2} \right) \hat{P}_1(x, z; t) = 0, \forall x, \forall z \leq 0, \forall t \quad (1)$$

Conditions at the boundary (interface F1 / F2) $z = 0$ (equal sound pressures normal and peculiar velocities in each medium) are written as follows:

$$\hat{p}_1(x, z; t) = \hat{p}_2(x, z; t), \quad \forall x, z = 0, \forall t \quad (2-a)$$

$$\hat{v}_1(x, z; t) \cdot \vec{n} = \hat{v}_2(x, z; t) \cdot \vec{n}, \quad \forall x, z = 0, \forall t \quad (2-b)$$

Where \vec{n} is the normal to the interface (here $\vec{n} = \vec{e}_z$).

The projection on the normal \vec{n} of Euler's equation outside sources in the medium F1 is written:

$$\rho_1 \frac{\partial \hat{v}_1}{\partial t} \cdot \vec{n} + \frac{\partial \hat{p}_1}{\partial n} = 0 \quad (3)$$

Because the projection operators on scalar \vec{n} and $\partial/\partial t$ commute,

$$\rho_1 \frac{\partial (\hat{v}_1 \cdot \vec{n})}{\partial t} + \frac{\partial \hat{p}_1}{\partial n} = 0 \quad (4-a)$$

Similarly, in the middle F2,

$$\rho_2 \frac{\partial (\hat{v}_2 \cdot \vec{n})}{\partial t} + \frac{\partial \hat{p}_2}{\partial n} = 0 \quad (4-b)$$

By using equations (4), the boundary condition (2-b) can be written, equivalently

$$\frac{1}{\rho_1} \frac{\partial \hat{p}_1}{\partial n}(x, z; t) = \frac{1}{\rho_2} \frac{\partial \hat{p}_2}{\partial n}(x, z; t), \quad \forall x, z = 0, \forall t \quad (5-a)$$

Or

$$\frac{1}{\rho_1} \frac{\partial \hat{p}_1}{\partial z}(x, z; t) = \frac{1}{\rho_2} \frac{\partial \hat{p}_2}{\partial z}(x, z; t), \quad \forall x, z = 0, \forall t \quad (5-b)$$

The reflected wave propagates to infinity (note that this is not a condition of Sommerfeld for the existing field at infinity).

2. 2. Writing the problem in the middle F2

The propagation equation in the fluid medium is written as F2

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c_2^2} \frac{\partial^2}{\partial t^2} \right) \hat{P}_2(x, z; t) = 0, \forall x, \forall z \geq 0, \forall t \quad (6)$$

The boundary conditions at the interface $z = 0$ are identical to those written in §. 2. 1. The medium F2 containing neither source nor border other than at $z = 0$, write a condition of no return.

3. Solutions of the problem

The pressure field \hat{P}_1 in the middle F1 is the sum of the pressure field incident \hat{P}_a and \hat{P}_b reflected pressure field can be written as follows

$$\hat{P}_1(x, z; t) = \hat{P}_a(x, z; t) + \hat{P}_b(x, z; t) = \hat{f}_1 \left(t - \frac{\vec{n}_a \cdot \vec{r}}{c_1} \right) + \hat{g}_1 \left(t - \frac{\vec{n}_b \cdot \vec{r}}{c_1} \right)$$

(7)

And the field F2 is written in the middle

$$\hat{P}_2(x, z; t) = \hat{f}_2 \left(t - \frac{\vec{n}_2 \cdot \vec{r}}{c_2} \right)$$

(8)

Where \vec{n}_a, \vec{n}_b and \vec{n}_2 denote the conditions of wave propagation incident, reflected and transmitted and \vec{r} denotes the position vector $\vec{OM} = x\vec{e}_x + z\vec{e}_z$.

The condition (2-a) equal pressures \hat{P}_1 and \hat{P}_2 at the

interface $z = 0$, whatever are the values of variables.

Following the functions \hat{f}_1, \hat{g}_1 and \hat{f}_2 which are identical, and their arguments :

$$t - \frac{\vec{n}_a \cdot \vec{r}}{c_1} = t - \frac{\vec{n}_b \cdot \vec{r}}{c_1} = t - \frac{\vec{n}_2 \cdot \vec{r}}{c_2}, \forall x, z = 0, \forall t \quad (9-a)$$

Or, noting n_{xa}, n_{xb} and n_{x2} the respective components on the axis (Ox) vectors n_a, n_b and n_2 ,

$$\frac{n_{xa}x}{c_1} = \frac{n_{xb}x}{c_1} = \frac{n_{x2}x}{c_2}, \forall x, z = 0, \forall t \quad (9-b)$$

This implies

$$\frac{n_{xa}}{c_1} = \frac{n_{xb}}{c_1} = \frac{n_{x2}}{c_2} \quad (9-c)$$

By expressing the components n_{xa}, n_{xb} and n_{x2} according to angles θ_1, θ'_1 and θ_2 , equations (9-c) are written

$$\frac{\sin \theta_1}{c_1} = \frac{\sin \theta'_1}{c_1} = \frac{\sin \theta_2}{c_2} \quad (10)$$

As a result, since θ_1 and θ'_1 are between 0 and $\pi/2$,

$$\theta_1 = \theta'_1 \quad (11)$$

They are reflecting equal angles of incidence and reflection. It should be noted firstly that the establishment of Snell's laws do not require the assumption of monochromatic wave, and secondly it is the conservation of the projection vector propagation directions on interface that involves laws of Snell, and not vice versa.

3. 1. Slowness surface

Slowness surface (L) is the location of the ends of the vector $\vec{m} = \vec{n}/C$ led to a fixed point O . Since \vec{m} and \vec{C} are collinear and $mC = 1$, surface area and slow speeds to match in reverse pole O and power 1. Slowness surface, analogous to the surface of optical indices, plays an important role in the problems of reflection and refraction. The slow surface of a material own supplies for any direction, the solutions of the wave equation. Accordingly, since the vector of the incident wave is known, simply add the Snell's law to the surfaces of delays for the two materials without calculation, the vectors of the waves could propagate in the one another, their polarization and acoustic rays (that is to say, the direction of energy propagation).

In fluid medium, the speed is the same for any direction of propagation, the slow surface is a sphere and its intersection with the plane of incidence (Oxz) here is a circle.

Case $c_1 < c_2$:

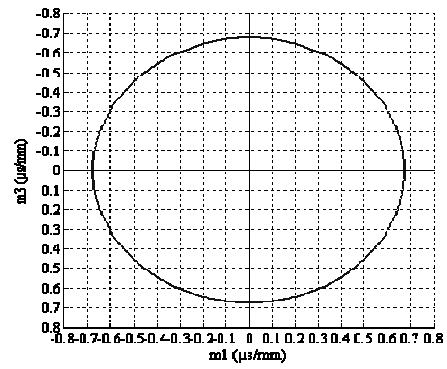


Figure 2. Slowness surface case $c_1 < c_2$.

Noting that each term of equation (10) corresponds to the projection on the x -axis vector slowness of the incident reflected and transmitted waves, using the intersection with the plane of incidence, surface delays of each of semi-infinite F1 and F2, can view, graphically some properties of waves propagating in each medium. It suffices to draw a line parallel to the axis Oz corresponding to the angle of incidence for θ_1 on the axis Ox , the amount $\frac{\sin \theta_1}{c_1}$

(Figure 3. Where $c_1 < c_2$ so $\frac{1}{c_1} > \frac{1}{c_2}$). Delaying this amount to the right provides an intersection with each curve of slow circles F1 and F2 and consequently, yields the angles of reflection and transmission.

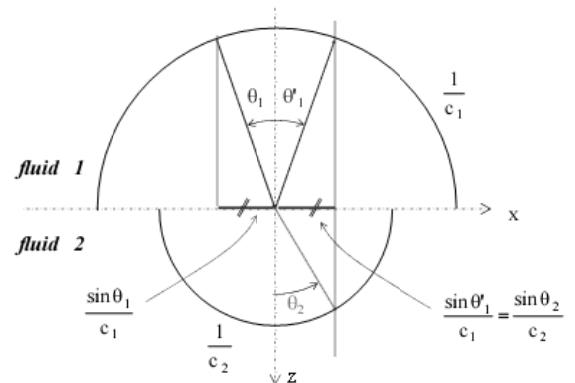


Figure 3. Using the intersection with the plane of incidence of the slowness surface of each semi-infinite medium [3].

3. 2. Harmonic solution

The source to infinity is monochromatic pulsation ω , a vector wave number associated with each plane wave pressure \hat{P}_a, \hat{P}_b and \hat{P}_2 can be defined, Are

$$\vec{k}_a = k_1 \vec{n}_a = \frac{\omega}{c_1} \vec{n}_a \quad (12-a)$$

$$\vec{k}_b = k_1 \vec{n}_b = \frac{\omega}{c_1} \vec{n}_b \quad (12-b)$$

$$\vec{k}_2 = k_2 \vec{n}_2 = \frac{\omega}{c_2} \vec{n}_2 \quad (12-c)$$

And the use of a solution can be written

$$\left(\widehat{p}_i(x, z; t) = \widehat{A}_i \exp\left\{-i\left(\vec{k}_i \cdot \overline{OM} - \omega t\right)\right\} \right)$$

$$\widehat{p}_a(x, z; t) = \widehat{A}_1 \exp\left\{-i\left(\vec{k}_a \cdot \overline{OM} - \omega t\right)\right\} \quad (13-a)$$

$$\widehat{p}_b(x, z; t) = \widehat{B}_1 \exp\left\{-i\left(\vec{k}_b \cdot \overline{OM} - \omega t\right)\right\} \quad (13-b)$$

And

$$\widehat{p}_2(x, z; t) = \widehat{A}_2 \exp\left\{-i\left(\vec{k}_2 \cdot \overline{OM} - \omega t\right)\right\} \quad (13-c)$$

According to equations (9-c) written in paragraph (§ 3) that come from writing the boundary conditions at the interface $z = 0$, written for all values of x and t , the projections on the interface (Ox) vector wave number \vec{k}_a, \vec{k}_b and \vec{k}_2 are equal :

$$k_{xa} = k_{xb} = k_{x2} = k_x \quad (14)$$

The laws of Snell (10) and equal angles of incidence and reflection (11) who sell require, in the F1 environment, equality (at sign) projections on the axis (Oz) vector wave number \vec{k}_a and \vec{k}_b , :

$$k_{za} = -k_{zb} = k_{z1} \quad (15)$$

In summary we write :

$$\vec{k}_a = k_x \vec{e}_x + k_{z1} \vec{e}_z \quad (16-a)$$

$$\vec{k}_b = k_x \vec{e}_x - k_{z1} \vec{e}_z \quad (16-b)$$

$$\vec{k}_2 = k_x \vec{e}_x + k_{z2} \vec{e}_z \quad (16-c)$$

And

$$\overline{OM} = x \vec{e}_x + z \vec{e}_z \quad (16-d)$$

The formula for the pressure field in the middle F1

$$\widehat{p}_1(x, z; t) = \widehat{p}_a(x, z; t) + \widehat{p}_b(x, z; t) = \widehat{A} e^{-i(k_x x + k_{z1} z - \omega t)} + \widehat{B} e^{-i(k_x x - k_{z1} z - \omega t)} \quad (17)$$

And that F2 is in the middle

$$\widehat{p}_2(x, z; t) = \widehat{A}_2 e^{-i(k_x x + k_{z2} z - \omega t)} \quad (18)$$

Equality of pressures (2-a) $z = 0$ leads to

$$\widehat{A}_1 e^{-i(k_x x - \omega t)} + \widehat{B}_1 e^{-i(k_x x - \omega t)} = \widehat{A}_2 e^{-i(k_x x - \omega t)}$$

Hence

$$\widehat{A}_1 + \widehat{B}_1 = \widehat{A}_2 \quad (19-a)$$

Or

$$-\widehat{R}_p + \widehat{T}_p = 1 \quad (19-b)$$

$$\widehat{R}_p = \widehat{B}_1 / \widehat{A}_1 \quad (20-a)$$

$$\widehat{T}_p = \widehat{A}_2 / \widehat{A}_1 \quad (20-b)$$

Are the coefficients of reflection and transmission of pressure amplitude.

The partial derivatives with respect to z pressures can be written

$$\frac{\partial \widehat{p}_1}{\partial z}(x, z; t) = ik_{z1} \left\{ -\widehat{A}_1 e^{-ik_{z1}z} + \widehat{B}_1 e^{ik_{z1}z} \right\} e^{-i(k_x x - \omega t)} \quad (21-a)$$

And

$$\frac{\partial \widehat{p}_2}{\partial z}(x, z; t) = -ik_{z2} \widehat{A}_2 e^{-ik_{z2}z} e^{-i(k_x x - \omega t)} \quad (21-b)$$

Their report in the condition (4-b) equality normal speeds (4-b) $z = 0$ leads to

$$\frac{ik_{z1}}{\rho_1} \left(-\widehat{A}_1 + \widehat{B}_1 \right) e^{-i(k_x x - \omega t)} = -\frac{ik_{z2}}{\rho_2} \widehat{A}_2 e^{-i(k_x x - \omega t)}, \forall x, z = 0, \forall t$$

Or

$$\frac{k_{z1}}{\rho_1} \left(-\widehat{A}_1 + \widehat{B}_1 \right) = -\frac{k_{z2}}{\rho_2} \widehat{A}_2 \quad (22-a)$$

Hence

$$\frac{k_{z1}}{\rho_1} \widehat{R}_p + \frac{k_{z2}}{\rho_2} \widehat{T}_p = \frac{k_{z1}}{\rho_1} \quad (22-b)$$

Solving the system of two equations (19) and (22) with two unknowns \widehat{R}_p and \widehat{T}_p leads to

$$\widehat{R}_p = \frac{-k_{z2}/\rho_2 + k_{z1}/\rho_1}{k_{z2}/\rho_2 + k_{z1}/\rho_1} \quad (23-a)$$

And

$$\widehat{T}_p = \frac{2k_{z1}/\rho_1}{k_{z2}/\rho_2 + k_{z1}/\rho_1} \quad (23-b)$$

By replacing k_{z1} and k_{z2} by their respective expressions in terms of k_1, θ_1, k_2 and θ_2

$$k_{z1} = k_1 \cos \theta_1 \quad (24-a)$$

And

$$k_{z2} = k_2 \cos \theta_2 \quad (24-b)$$

And introducing the characteristic impedances Z_1 and Z_2 of the two media F1 and F2

$$Z_1 = \rho_1 c_1 \quad (25-a)$$

And

$$Z_2 = \rho_2 c_2 \quad (25-b)$$

The coefficients of reflection and transmission (23) can be written

$$\widehat{R}_p = \frac{-\cos \theta_2 / Z_2 + \cos \theta_1 / Z_1}{\cos \theta_2 / Z_2 + \cos \theta_1 / Z_1} \quad (26-a)$$

And

$$\widehat{T}_p = \frac{2 \cos \theta_1 / Z_1}{\cos \theta_2 / Z_2 + \cos \theta_1 / Z_1} \quad (26-b)$$

Or

$$\widehat{R}_p = \frac{Z_2 / \cos \theta_2 - Z_1 / \cos \theta_1}{Z_2 / \cos \theta_2 + Z_1 / \cos \theta_1} \quad (27-a)$$

And

$$\widehat{T}_p = \frac{2Z_2 / \cos \theta_2}{Z_2 / \cos \theta_2 + Z_1 / \cos \theta_1} \quad (27-b)$$

Examples of changes in coefficients of reflection and transmission of pressure amplitude, depending on the angle of incidence are presented in the figures below.

For an angle of incidence equal to 30° [1, 2, 3], a "rupture" appears in these curves. This angle is the critical angle for the interface considered above which the transmitted waves are evanescent.

3.3. Evanescent waves

In the case where $c_1 > c_2$ and $\theta_1 > \theta_c$, the transmitted wave becomes evanescent: its amplitude decreases exponentially with a distance from the interface [1, 2, 3]. The total reflection phase shift accompanied \mathcal{X} by a reflection given by the argument of the complex reflection coefficient. The sum of the incident wave and the reflected wave produces a standing wave totally vertically and horizontally progressive.

4. Numerical results

Example 1:

$$\rho_1 = 2000 \text{ Kg/m}^3, c_1 = 750 \text{ m/s}, \rho_2 = 2500 \text{ Kg/m}^3, c_2 = 1500 \text{ m/s}$$

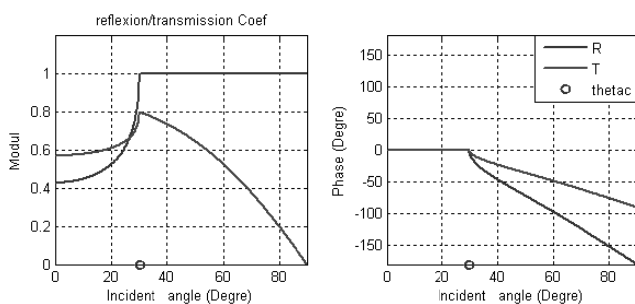


Figure 4. Magnitude and phase of the reflection and transmission coefficient for Example 1

Example 2:

$$\rho_1 = 3000 \text{ Kg/m}^3, c_1 = 750 \text{ m/s}, \rho_2 = 1000 \text{ Kg/m}^3, c_2 = 1500 \text{ m/s}$$

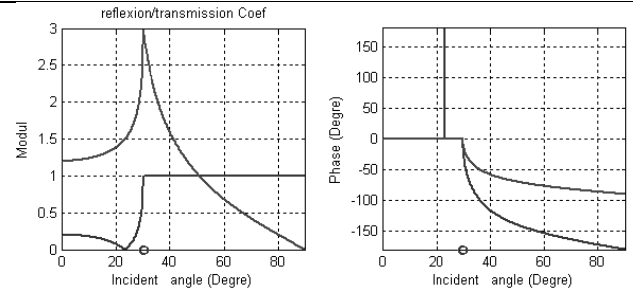


Figure 5. Magnitude and phase of the reflection and transmission coefficient for Example 2

Example 3:

$$\rho_1 = 1000 \text{ Kg/m}^3, c_1 = 750 \text{ m/s}, \rho_2 = 1000 \text{ Kg/m}^3, c_2 = 1500 \text{ m/s}$$

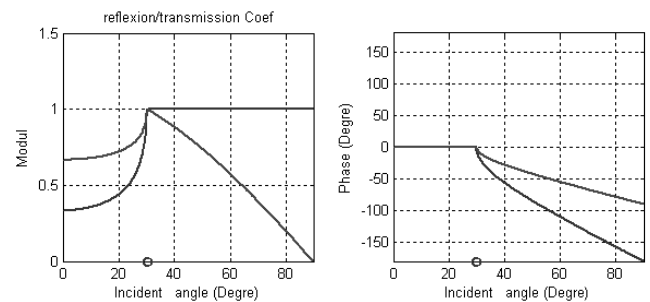


Figure 6. Magnitude and phase of the reflection and transmission coefficient for Example 3.

With :

R : Reflection coefficient;

T : Transmission coefficient;

Using the construction of Figure 3, it appears that from a certain angle of incidence noted θ_c and called critical angle for the interface F1/F2 considered (where $c_1 < c_2$) [1, 2, 3], there is no longer intersects the line parallel to Oz with the slow curve of the medium (Figure 7-a). Corresponding waves in the middle F2 are no longer propagating and become evanescent: they see their energy propagating in parallel to the interface whereas the pressure amplitude decreases exponentially in the direction of z increasing.

The value of the critical angle θ_c is given by :

$$\theta_c = \arcsin \frac{c_1}{c_2}.$$

This corresponds to $\theta_c = \pi/2$.

When $c_1 > c_2$, construction of Figure 7-c. shows that there is no critical angle on the interface considered.

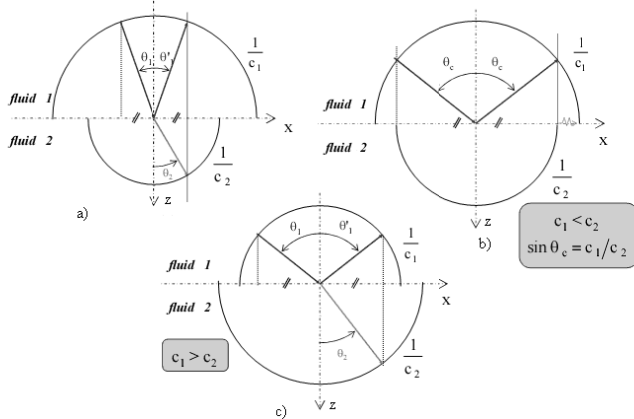


Figure 7. Intersection with the plane of incidence of the slowness surface for an interface F1/F2 -a) $c_1 < c_2$, -b)

$$c_1 < c_2, - c) c_1 > c_2 [3].$$

4. 1. Discussion of results

In the ideal case where the interface is considered perfectly flat, while the second medium has characteristics different from those acoustic (ρ_1, c_1) and (ρ_2, c_2) , a portion of the energy of the incident wave will be transmitted in the second medium. The consequence is that the reflected wave will be assigned a reflection coefficient of modulus less than or equal to 1 [1, 2, 3, 4, 5, 6]. If the reflecting medium has a velocity greater than that of the F1 community, we have seen that there exists a critical angle θ_c at which transmission is impossible, the modulus of the reflection coefficient is then equal to 1 [1, 2, 3, 4, 5, 6] (the phenomenon of total reflection). At the critical angle the reflection coefficient decreases sharply with the angle, and when close to vertical it depends only on the characteristic impedances of two media. In the presence of damping in the second medium the phenomena will be slightly modified; in particular the coefficient of total reflection will be slightly less than 1. Finally, if the reflecting medium is impedance is very small or very large compared to that of the F1 community, the incident wave will reflect almost no energy loss, the reflection coefficient (the ratio of the amplitudes of waves reflected and incident) will equal 1

regardless of the angle [1, 2, 3].

The reflection coefficient of transmission of the pressure in the second medium, affecting the amplitude of the refracted wave is given by $T = 1 + R$. The pressure level of the transmitted wave can exceed that of the incident wave. This result simply reflects greater continuity of pressure on both sides of the interface, and in no way violates the law of conservation of energy and can easily show that the intensity is equal to the sum reflected and transmitted intensities.

5. Conclusion

When the speeds of wave propagation in c_1 and c_2 circles F1 and F2 are such that $c_1 < c_2$, there is an angle of incidence θ_1 rated θ_c called critical angle for this interface from which the transmitted wave in the middle F2 becomes evanescent. The reflectance in the middle F1 becomes equal to 1 module (total reflection), but there is always the presence of acoustic energy in the medium F2. The evanescent transmitted wave propagates parallel to the interface, while its amplitude decreases exponentially with depth (when z increases), perpendicular to the interface.

References

[1] L. E. Kinsler, A. R. Frey, A. B. Coppens, J. V. Sanders, Fourth edition, John Wiley & Sons, Inc, New york. 548 pages, ISBN 0-471-84789-5.
 [2] E. S. Krebs and P. F. Daley, Geophys. J. Int, 170, 205-216, 2007.
 [3] C. Potel, M. Bruneau, Ed. Ellipse collection Technosup, 352 pages, 2006.
 [4] C. Potel, J.F. De Belleval, J. of Applied Physics, 77, 12, 6152-6161, 1995.
 [5] C. Potel, J.F. De Belleval, E. Genay, Ph. Gagniol, Acustica-Acta Acustica, 82, 5, 738-748, 1996.
 [6] C. Potel, S. Devolder, A.U. Rehman, J.F. De Belleval, J.M. Gherbezza, O. Leroy, M. Wevers, J. Appl. Phys., 86, 2, 1128-1135, 1999.