Temperature effect on the vibration characteristics of carbon nanotubes

B. Kheroubi, H. Heireche, A. Benzair and A. Tounsi

a Département of Physic, University of Sidi Bel Abbès, Algérie
b Materials and Hydrology Laboratory, University of Sidi Bel Abbès, Algérie

Received: 30 April 2014, accepted 26 May 2014

Abstract

In this work, the thermal buckling properties of carbon nanotube with small scale effect are studied. Based on the nonlocal continuum theory and the Timoshenko beam model, the governing equation is derived and the critical buckling temperature is presented. The influences of the scale coefficients, the ratio of the length to the diameter, the transverse shear deformation and rotary inertia are discussed. It can be observed that the small scale effects are significant and should be considered for thermal analysis of carbon nanotube. The critical buckling temperature becomes higher with the ratio of length to the diameter increasing. Furthermore, for smaller ratios of the length to the diameter and higher mode numbers, the transverse shear deformation and rotary inertia have remarkable influences on the thermal buckling behaviors.

Keywords: carbon nanotube, scale effects, nondimensional critical buckling temperature, thermal buckling, Timoshenko beam.

PACS: 1. Introduction

Since their discovery, carbon nanotubes were quickly imposed in various areas, particularly in the area of nanotechnology. The unique structure of this compound derived graphene makes it one of the most interesting both electronic and mechanical perspective and promises many marketable outs.

Carbon nanotubes were discovered by chance among the soot obtained in the discharge of an electric arc between two carbon electrodes. It is in this type of experience as researchers from the Universities of Heidelberg and Houston had observed in 1986 carbon molecules of a new type: the fullerenes.

It is in these Sumio Lijima that soot Japanese scholar observed in 1991 funny little hollow tubes made of carbon atoms and that no one had previously noticed. They were given the name of ‘carbon nanotubes’.

Some researchers have shown that the mechanical behaviors of the carbon nanotube are sensitive to the thermal effects in the external environment [24]. Recently, considering the effects of the transverse shear deformation and rotary inertia,

Hsu et al. [5] and Lee and Chang [6] have been undertaken to study the thermal buckling properties of the carbon nanotubes using Timoshenko beam model. At the end to see the scale effect (e,a) on the buckling properties of carbon nanotubes, equation involving the nonlocal Timoshenko beam theory has been established.

2. Motion equation

The nonlocal elasticity model was first presented by Eringen (1972). According to this model, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body. The constitutive equation of the nonlocal elasticity can be written as follows

\[
[1-(e_0 a)^2 \nabla^2] \sigma_{ij} = C_{ij} \varepsilon_{kl} \quad (2.1)
\]

Where \( C_{ij} \) is the elastic modulus tensor of the classical isotropic elasticity; and \( \sigma_i \) and \( \varepsilon_i \) are the stress and strain tensors, respectively. In addition, \( e_0 \) is a nondimensional material constant, determined by experiments, and \( a \) is an internal characteristic length (e.g. lattice parameter, granular distance). Therefore, \( e_0 a \) is a constant parameter showing the small-scale effect in nano-structures.

For the nonlocal Timoshenko beam theory, the Hook’s law of carbon nanotube can be expressed as the following partial differential forms:

\[
\sigma_x = (e_0 a)^2 \frac{\partial^2 \varepsilon_x}{\partial x^2} = E \varepsilon_x \quad (2.2a)
\]

\[
\tau_{x,z} = (e_0 a)^2 \frac{\partial^2 \tau_{x,z}}{\partial x^2} - G \gamma_{xz} \quad (2.2b)
\]

Where:

- \( \sigma_x \) : The axial stress,
- \( \tau_{x,z} \) : The shear stress,
Temperature effect on the vibration characteristics of carbon nanotubes

\[ \varepsilon_x = z \frac{\partial \phi}{\partial x}, \quad (2.3a) \]
\[ \gamma_{xx} = \frac{\partial w}{\partial x} - \psi, \quad (2.3b) \]

Where \( W \) is the transverse displacement and \( \psi \) the rotation caused by bending.

For the Timoshenko beam model with the thermal stress, the following relation can be derived:

\[ \frac{\partial S}{\partial x} = -N_T^2 \frac{\partial^2 W}{\partial x^2}, \quad (2.4a) \]
\[ \frac{\partial M}{\partial x} + S - 0, \quad (2.4b) \]

Where \( S \) is the shear force, \( M \) the resultant bending moment and \( N_T \) the thermal moment which can be expressed as:

\[ N_T = -\frac{\varepsilon_a T A_c}{1-\nu}, \quad (2.5) \]

Where \( \nu \) is the thermal expansion coefficient, \( T \) the temperature change, \( A_c \) the cross area and \( \nu \) the Poisson’s ratio.

The bending moment and the shear force can be defined by

\[ M = \int_{A_c} z \sigma_z dA_c, \quad (2.6a) \]
\[ S = K \int_{A_c} \gamma_{xx} dA_c, \quad (2.6b) \]

\[ \frac{\partial S}{\partial x} = -N_T^2 \frac{\partial^2 W}{\partial x^2} \]
\[ \frac{\partial M}{\partial x} + S - 0 \]

Substituting equations (2.2a), (2.5a), we obtain:

\[ \int_{A_c} z \sigma_z dA_c - (e_0 a)^2 \int_{A_c} z \frac{\partial^2 \sigma_z}{\partial x^2} dA_c = E \int_{A_c} \varepsilon_x dA_c \]

Substituting equations (2.3a), (2.5a), we obtain:

\[ \int_{A_c} z \sigma_z dA_c - (e_0 a)^2 \int_{A_c} z \frac{\partial^2 \sigma_z}{\partial x^2} dA_c = E \int_{A_c} z \frac{\partial \psi}{\partial x} dA_c \]

By integrating equation (2.1b), one obtains:

\[ \int_{A_c} \gamma_{xx} dA_c - (e_0 a)^2 \int_{A_c} \frac{\partial^2 \tau_{xx}}{\partial x^2} dA_c = \int_{A_c} G \gamma_{xx} dA_c \]

Substituting equation (2.5b), we obtain:

\[ S = (e_0 a)^2 \frac{\partial^2 S}{\partial x^2} = K a G \left( \frac{\partial W}{\partial x} - \psi \right) \]

\[ (2.7b) \]

Where \( I = \int_{A_c} z^2 dA_c \) is the moment of inertia and \( K \) the shear correction factor which is used to compensate for the error due to the constant shear stress assumption.

Based on Eqs. (2.3a) and (2.6a) the following relation can be obtained: \( (2.3b) \)

\[ M = E I \frac{\partial \psi}{\partial x} + (e_0 a)^2 \left( -\frac{\partial S}{\partial x} \right). \]

Substituting Eq. (2.3a) into Eq. (2.7), we can obtain

\[ M = E I \frac{\partial \psi}{\partial x} + (e_0 a)^2 \left( -N_T \frac{\partial^2 W}{\partial x^2} \right). \]

Based on Eqs. (2.3a) and (2.6b), it can be derived that

\[ S = K a G \left( \frac{\partial W}{\partial x} - \psi \right) + (e_0 a)^2 \left( -N_T \frac{\partial^2 W}{\partial x^2} \right). \]

Substituting Eq. (2.9) into Eq. (2.3a), we can obtain

\[ K a G \left( \frac{\partial^2 W}{\partial x^2} - \frac{\partial \psi}{\partial x} \right) + (e_0 a)^2 \left( -N_T \frac{\partial^2 W}{\partial x^2} \right) = -N_T \frac{\partial^2 W}{\partial x^2}. \]

Based on Eqs. (2.3b), (2.8) and (2.9), the following relation can be derived:

\[ E I \frac{\partial^2 \psi}{\partial x^2} + K a G \left( \frac{\partial W}{\partial x} - \psi \right) = 0. \]

It can be observed that Eqs. (2.10) and (2.11) are the governing equations. For the hinged boundary condition, the solution of carbon nanotube can be expressed as

\[ w = W \sin(\lambda x), \]
\[ \psi = \psi \cos(\lambda x), \]

Where \( W \) and \( \psi \) are the amplitudes of the deflection and the slope, \( \lambda = k \pi / L \) and \( k \) a positive integer which is related to the buckling modes.

Substituting Eq. (2.12) into Eqs. (2.10) and (2.11), we can obtain

\[ (e_0 a)^2 \lambda^2 + 1) \lambda^2 N_T W + \lambda^2 K a G W \]

\[ \lambda K a G \psi = 0, \]

(2.14a)

\[ \lambda K a G W - (\lambda^2 E I + K a G) \psi = 0. \]

(2.14b)

Then, the critical temperature with the nonlocal continuum theory can be derived as

\[ T_{cr}^{non} = \frac{\lambda^2 K G (1-\nu)}{a(e_0 a)^2 \lambda^2 + 1)(K a G + E I \lambda^2)} \]

Substituting the critical buckling temperature can be expressed as the following form [8-9]:

\[ P_{cr} = \frac{T_{cr}^{non}}{f_{nh} k^2} \]
3. Results and discussions

In this section, numerical calculations for the thermal buckling properties of carbon nanotube are carried out. The material constants used in the calculation are the Young’s modulus $E=1$ TPa, the Poisson’s ratio $\nu=0.3$, the shear modulus $G=E/[2(1+\nu)]$, the shear coefficient $k = 10/9$ and the temperature expansion coefficient $\alpha = 1.1 \times 10^{-6}$ K$^{-1}$ which is for the case of the high temperature [10, 11, 12, 13]. It should be noted that according to the previous discussions about the values of $e_0$ and $a$ in detail, $e_0a$ is usually considered as the single scale coefficient which is smaller than 2.0 nm for nanostructure[14,15].

![Fig.1](image1.png)  
**Fig.1.** Relation between the critical buckling temperature ($P$) and the mode number ($k$) with different scale coefficients ($e_0a$), the value of $L/d=20$.

The relation between the critical temperature ($P$) and the mode number ($k$) is presented in fig. 1. The ratio of the length to the diameter, $L/d$ is 20. The scale coefficients $e_0a = 0$, 1 and 2 nm are considered. The most notable feature is that the results based on the two theories are almost the same for small mode numbers. However, the difference becomes obvious with the mode number increasing. The classical elastic (i.e. the local) model, which does not consider the small scale effects, will give a higher approximation for the critical buckling temperature. But the nonlocal continuum theory will present an accurate and reliable result.

![Fig.2](image2.png)  
**Fig.2.** Relation between the critical buckling temperature ($P$) and the mode number ($k$) with different values of $L/d$. The scale coefficient $e_0a = 1$nm.

The influences of the ratio of the length to the diameter ($L/d$) on the critical buckling temperature are shown in fig. 2. The scale coefficient is 1 nm. From fig. 2, it can be seen that when the mode number is less than 4, the difference is not obvious. When the mode number is larger than 5, this influence becomes remarkable. Moreover, the critical buckling temperatures for all of the three ratios become larger with the mode number increasing. The larger the ratio of the length to the diameter is, the higher the nondimensional critical buckling temperature becomes. It means that the ratio of the length to the diameter has significant influence on the critical buckling temperature for larger mode numbers.

![Fig.3a](image3.png)  
**Fig.3a.** Relation between the critical buckling temperature ($P$) and the value of $L/d$ with different scale coefficients ($e_0a$), $k = 1$. 
The relation between the critical buckling temperature and the ratio of the length to the diameter are shown in fig. 3(a)–(c). The scale coefficients $e_0a = 0$, 1 and 2 nm and the mode number $k = 1$, 5 and 10 are considered, respectively. It can be seen that the ranges of the critical buckling temperature for these mode numbers are quite different. In fig. 3(a), the range is the smallest for $k = 1$, but the range is the largest for $k = 10$ in fig. 3(c). It means that the larger the mode number is, the higher the critical buckling temperature becomes.

Furthermore, it can be observed that when the ratio of the length to the diameter is small, the scale effects are significant. However, the scale effects on the nondimensional critical buckling temperature will diminish with the ratio (i.e. $L/d$) increasing. It implies that the scale effects on the thermal buckling properties are not obvious for slender carbon nanotube but should be taken into account for short nanotube.

4. Conclusions

In this work, based on the nonlocal continuum theory, the governing equation is presented and the critical buckling temperature of carbon nanotube is derived. The influences of the scale coefficient, the ratio of the length to the diameter, the transverse shear deformation and rotary inertia on the thermal buckling properties are discussed. From the results, it can be concluded that the small scale effects should be considered for the thermal buckling behaviors, especially for higher mode numbers and short carbon nanotube. The critical buckling temperature can be changed by different ratios of the length to the diameter. The influences of the transverse shear deformation and rotary inertia are obvious for higher mode numbers and smaller ratios of the length to the diameter.

References