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Temperature effect on the vibration characteristics of carbon nanotubes

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Abstract

In this work, the thermal buckling properties of carbon nanotube with small scale effect are studied. Based on the nonlocal continuum theory and the Timoshenko beam model, the governing equation is derived and the critical buckling temperature is presented. The influences of the scale coefficients, the ratio of the length to the diameter, the transverse shear deformation and rotary inertia are discussed. It can be observed that the small scale effects are significant and should be considered for thermal analysis of carbon nanotube. The critical buckling temperature becomes higher with the ratio of length to the diameter increasing. Furthermore, for smaller ratios of the length to the diameter and higher mode numbers, the transverse shear deformation and rotary inertia have remarkable influences on the thermal buckling behaviors.
Keywords: carbon nanotube, scale effects, nondimensional critical buckling temperature, thermal buckling, Timoshenko beam.
PACS:

1. Introduction

Since their discovery, carbon nanotubes were quickly imposed in various areas, particularly in the area of nanotechnology. The unique structure of this compound derived graphene makes it one of the most interesting both electronic and mechanical perspective and promises many marketable outs.

Carbon nanotubes were discovered by chance among the soot obtained in the discharge of an electric arc between two carbon electrodes. It is in this type of experience as researchers from the Universities of Heidelberg and Houston had observed in 1986 carbon molecules of a new type: the fullerenes.

It is in these Sumio Lijima that soot [1] Japanese scholar observed in 1991 funny little hollow tubes made of carbon atoms and that no one had previously noticed. They were given the name of "carbon nanotubes ".

Some researchers have shown that the mechanical behaviors of the carbon nanotube are sensitive to the thermal effects in the external environment [2-4]. Recently, considering the effects of the transverse shear deformation and rotary inertia,

Hsu et al. [5] and Lee and Chang [6] have been undertaken to study the thermal buckling properties of the carbon nanotubes using Timoshenko beam model. At the end to see the scale effect (e_0a) on the buckling properties of carbon nanotubes, equation involving the nonlocal Timoshenko beam theory has been established.

2. Motion equation

The nonlocal elasticity model was first presented by Eringen (1972). According to this model, the stress at a reference point in the body is dependent not only on the strain state at that point, but also on the strain state at all of the points throughout the body. The constitutive equation of the nonlocal elasticity can be written as follows

$$[1-(e_0a)^2\nabla^2]\,\sigma_{ij}= C_{ijk}\varepsilon_{kl} \qquad (2.1)$$

Where C_{ijkl} is the elastic modulus tensor of the classical isotropic elasticity; and σ_{ij} and ε_{kl} are the stress and strain tensors, respectively. In addition, e_0 is a nondimensional material constant, determined by experiments, and a is an internal characteristic length (e.g. a lattice parameter, granular distance). Therefore, e_0a is a constant parameter that showing the small-scale effect in nano-structures.

For the nonlocal Timoshenko beam theory, the Hook's law of carbon nanotube can be expressed as the following partial differential forms:

$$\sigma_x-(e_0a)^2\frac{\partial^2\sigma_x}{\partial x^2}=E\varepsilon_x\,, \qquad (2.2a)$$

$$\tau_{x,z}-(e_0a)^2\frac{\partial^2\tau_{xz}}{\partial x^2}=G\gamma_{xz}\,, \qquad (2.2b)$$

Where:
σ_x : The axial stress,
$\tau_{x,z}$: The shear stress,

ε_x : The axial strain,
γ_{xz} : The shear strain,
E: The Young's modulus,
G: The shear modulus,
e_0 : Scale effect coefficient.

The expressions of the axial strain and the shear strain are [7]:

$$\varepsilon_x = z\frac{\partial\psi}{\partial x}, \tag{2.3a}$$
$$\gamma_{xz} = \frac{\partial W}{\partial x} - \psi, \tag{2.3b}$$

Where W is the transverse displacement and ψ the rotation caused by bending.
For the Timoshenko beam model with the thermal stress, the following relation can be derived:

$$\frac{\partial S}{\partial x} = -N_T\frac{\partial^2 w}{\partial x^2}, \tag{2.4a}$$
$$\frac{\partial M}{\partial x} + S = 0, \tag{2.4b}$$

Where S is the shear force, M the resultant bending moment and N_T the thermal force which can be expressed as:

$$N_T = -\frac{E\alpha T A_c}{1-2\upsilon}, \tag{2.5}$$

Where α is the thermal expansion coefficient, T the temperature change, A_c the cross area and υ the Poisson's ratio.
The bending moment and the shear force can be defined by

$$M = \int_{A_c} z\sigma_x \mathrm{d}A_c, \tag{2.6a}$$
$$S = K\int_{A_c} \tau_{xz}\mathrm{d}A_c. \tag{2.6b}$$

Multiplying equation (2.1a) By z dA_c and integrating the result on the surface of A_c, we obtain:

$$\int_{A_c} z\sigma_x dA_c - (e_0a)^2\int_{A_c} z\frac{\partial^2\sigma_x}{\partial x^2}dA_c = E\int_{A_c} z\ \varepsilon_x\, dA_c$$

Substituting equations (2.2a), (2.5a), we obtain:

$$\int_{A_c} z\sigma_x dA_c - (e_0a)^2\int_{A_c} z\frac{\partial^2\sigma_x}{\partial x^2}dA_c = E\int_{A_c} z^2\frac{\partial\psi}{\partial x}\, dA_c$$

Avec: $I = \int_{A_c} z^2\mathrm{d}A_c, \frac{\partial^2 M}{\partial x^2} = \int_{A_c} z\frac{\partial^2\sigma_x}{\partial x^2}dA_c$
Or:

$$M - (e_0a)^2\frac{\partial^2 M}{\partial x^2} = EI\frac{\partial\psi}{\partial x}, \tag{2.7a}$$

By integrating equation (2.1b), one obtains:

$$\int_{A_c} \tau_{xz}dA_c - (e_0a)^2\int_{A_c}\frac{\partial^2\tau_{xz}}{\partial x^2}\mathrm{dA_c} = \int_{A_c} G\gamma_{xz}dA_c$$

Substuting equation (2.5b)

and $\frac{\partial^2 S}{\partial x^2} = \int_{A_c}\frac{\partial^2\tau_{xz}}{\partial x^2}\mathrm{dA_c}$
So we obtain:

$$S - (e_0a)^2\frac{\partial^2 S}{\partial x^2} = KA_cG\left(\frac{\partial W}{\partial x} - \psi\right) \tag{2.7b}$$

Where $I = \int_{A_c} z^2\mathrm{d}A_c$ is the moment of inertia and K the shear correction factor which is used to compensate for the error due to the constant shear stress assumption.

Based on Eqs. (2.3b) and (2.6a) the following relation can be obtained: (2.3b)

$$M = EI\frac{\partial\psi}{\partial x} + (e_0a)^2\left(-\frac{\partial S}{\partial x}\right). \tag{2.8}$$

Substituting Eq. (2.3a) into Eq. (2.7), we can obtain

$$M = EI\frac{\partial\psi}{\partial x} + (e_0a)^2\left(N_T\frac{\partial^2 W}{\partial x^2}\right). \tag{2.9}$$

Based on Eqs. ((2.3a) and (2.6b), it can be derived that

$$S = KA_cG\left(\frac{\partial W}{\partial x} - \psi\right) + (e_0a)^2\left(-N_T\frac{\partial^3 W}{\partial x^3}\right). \tag{2.10}$$

Substituting Eq. (2.9) into Eq. (2.3a), we can obtain

$$KA_cG\left(\frac{\partial^2 W}{\partial x^2} - \frac{\partial\psi}{\partial x}\right) + (e_0a)^2\left(-N_T\frac{\partial^4 W}{\partial x^4}\right) = -N_T\frac{\partial^2 W}{\partial x^2}. \tag{2.11}$$

Based on Eqs. (2.3b), (2.8) and (2.9), the following relation can be derived:

$$EI\frac{\partial^2\psi}{\partial x^2} + KA_cG\left(\frac{\partial W}{\partial x} - \psi\right) = 0. \tag{2.12}$$

It can be observed that Eqs. (2.10) and (2.11) are the governing equations. For the hinged boundary condition, the solution of carbon nanotube can be expressed as

$$w = W\sin(\lambda x),$$
$$\psi = \psi\cos(\lambda x), \tag{2.13}$$

Where W and ψ are the amplitudes of the deflection and the slope, $\lambda = k\pi/L$ and k a positive integer which is related to the buckling modes.

Substituting Eq. (2.12) into Eqs. ((2.10) and (2.11), we can obtain

$$[(e_0a)^2\lambda^2 + 1]\lambda^2 N_T W + \lambda^2 KA_cGW - \lambda KA_cG\psi = 0, \tag{2.14a}$$
$$\lambda KA_cGW - (\lambda^2 EI + KA_cG)\psi = 0. \tag{2.14b}$$

Then, the critical temperature with the nonlocal continuum theory can be derived as

$$T_{cr}^{non} = \frac{\lambda^2 KGI(1-2\upsilon)}{\alpha[(e_0a)^2\lambda^2+1](KA_cG+EI\lambda^2)} \tag{2.15}$$

As a result, the critical buckling temperature can be expressed as the following form [8-9]:

$$P_{cr} = \frac{\mathrm{T_{cr}^{non}}}{\mathrm{I/\alpha A_c L^2}}$$

3. Results and discussions

In this section, numerical calculations for the thermal buckling properties of carbon nanotube are carried out. The material constants used in the calculation are the Young's modulus E=1 TPa, the Poisson's ratio υ=0.3, the shear modulus G=E/[2(1+υ)], the shear coefficient k = 10/9 and the temperature expansion coefficient α =1.1× 10^{-6} K^{-1} which is for the case of the high temperature **[10, 11, 12, 13]**. It should be noted that according to the previous discussions about the values of e_0 and a in detail, e_0a is usually considered as the single scale coefficient which is smaller than 2.0 nm for nanostructure**[14,15]**.

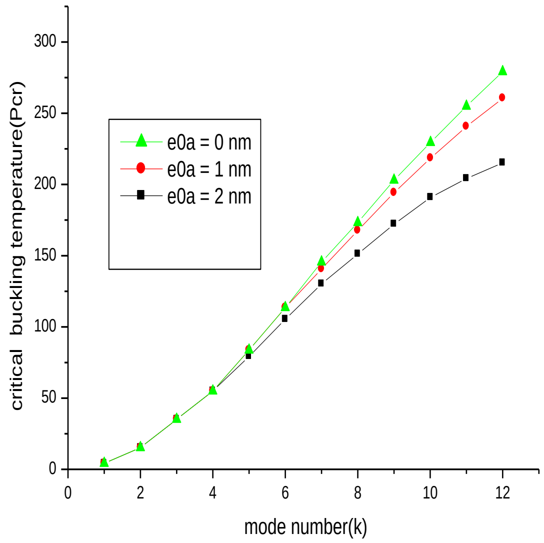

Fig.1. Relation between the critical buckling temperature (P_{cr}) and the mode number (k) with different scale coefficients (e_0a), the value of L/d=20.

The relation between the critical temperature (P_{cr}) and the mode number (k) is presented in fig. 1. The ratio of the length to the diameter, L/d is 20. The scale coefficients e_0a =0, 1 and 2 nm are considered. The most notable feature is that the results based on the two theories are almost the same for small mode numbers. However, the difference becomes obvious with the mode number increasing. The classical elastic (i.e. the local) model, which does not consider the small scale effects, will give a higher approximation for the critical buckling temperature. But the nonlocal continuum theory will present an accurate and reliable result.

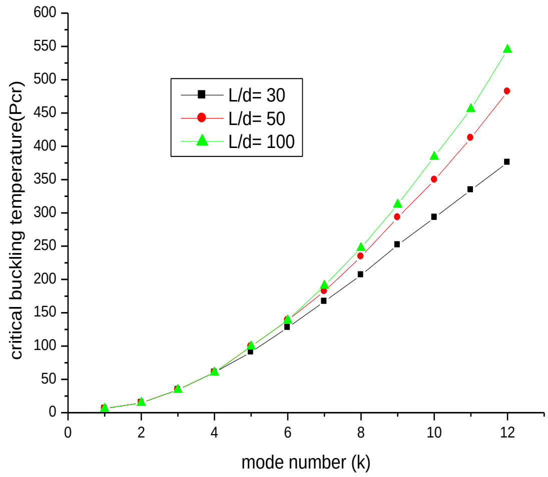

Fig.2. Relation between the critical buckling temperature (P_{cr}) and the mode number (k) with different values of L/d. The scale coefficient e_0a =1nm.

The influences of the ratio of the length to the diameter (L/d) on the critical buckling temperature are shown in fig. 2. The scale coefficient is 1 nm. From fig. 2, it can be seen that when the mode number is less than 4, the difference is not obvious. When the mode number is larger than 5, this influence becomes remarkable. Moreover, the critical buckling temperatures for all of the three ratios become larger with the mode number increasing. The larger the ratio of the length to the diameter is, the higher the nondimensional critical buckling temperature becomes. It means that the ratio of the length to the diameter has significant influence on the critical buckling temperature for larger mode numbers.

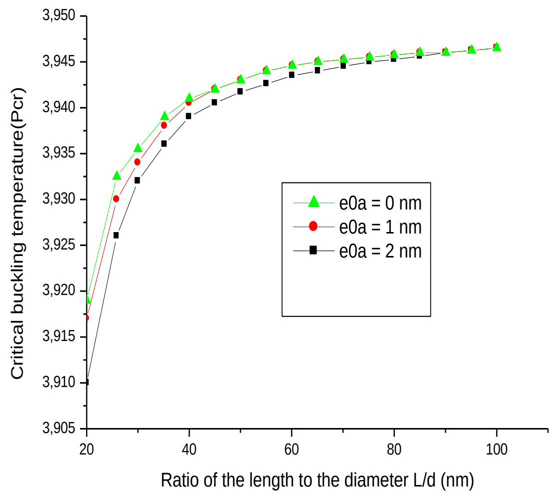

Fig.3.a. Relation between the critical buckling temperature (P_{cr}) and the value of L/d with different scale coefficients (e_0a), k = 1.

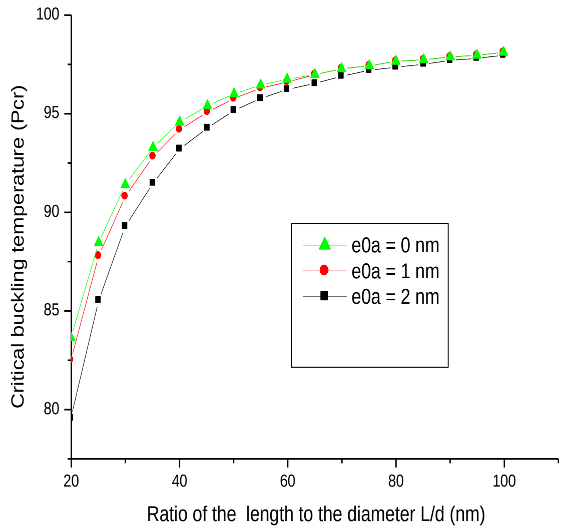

Fig.3.b. Relation between the critical buckling temperature (P_{cr}) and the value of L/d with different scale coefficients (e_0a), k = 5.

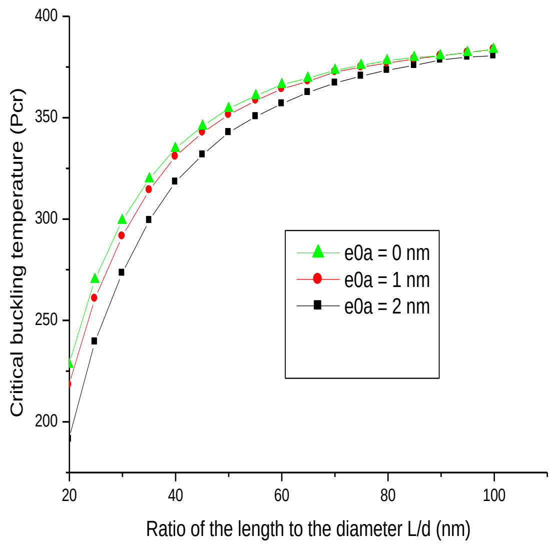

Fig.3.c. Relation between the critical buckling temperature (P_{cr}) and the value of L/d with different scale coefficients (e_0a), k = 10.

The relation between the critical buckling temperature and the ratio of the length to the diameter are shown in fig. 3(a)–(c). The scale coefficients e_0a = 0, 1 and 2 nm and the mode number k = 1, 5 and 10 are considered, respectively. It can be seen that the ranges of the critical buckling temperature for these mode numbers are quite different. In fig. 3(a), the range is the smallest for k = 1, but the range is the largest for k = 10 in fig. 3(c). It means that the larger the mode number is, the higher the critical buckling temperature becomes.

Furthermore, it can be observed that when the ratio of the length to the diameter is small, the scale effects are significant. However, the scale effects on the nondimensional critical buckling temperature will diminish with the ratio (i.e. L/d) increasing. It implies that the scale effects on the thermal buckling properties are not obvious for slender carbon nanotube but should be taken into account for short nanotube.

4. Conclusions

In this work, based on the nonlocal continuum theory, the governing equation is presented and the critical buckling temperature of carbon nanotube is derived. The influences of the scale coefficient, the ratio of the length to the diameter, the transverse shear deformation and rotary inertia on the thermal buckling properties are discussed. From the results, it can be concluded that the small scale effects should be considered for the thermal buckling behaviors, especially for higher mode numbers and short carbon nanotube. The critical buckling temperature can be changed by different ratios of the length to the diameter. The influences of the transverse shear deformation and rotary inertia are obvious for higher mode numbers and smaller ratios of the length to the diameter.

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