Analytical evaluation of criticality in RZ9 reactor

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ABSTRACT

The diffusion equation is widely used to estimate the reactor properties. In this work, this method is presented to evaluate the effective multiplication factor of neutrons in Oklo natural nuclear reactors. The results obtained by analysis of the geometric configurations as function of the reactor’s dimensions, are compared with those obtained using the Monte Carlo simulation method for the same configurations of Oklo natural nuclear reactor.

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1. Introduction

Natural Nuclear fission reactors are very high grade Uranium ores where chain fission reactions took place spontaneously 2 billion years ago and have been sustained for quite a long time in a natural environment without any human contribution [1,2].

The possibility of natural fission reactors occurring was predicted by P.Kuroda in 1956 following a suggestion by G. Whetherill and M. Inghram [3], but no trace of a natural reactor had been found. The existence of these reactors was discovered in 1972 at Oklo in Gabon (West Africa) and known as “The OKLO phenomenon”. To date, about fifteen natural fission reactors have been unearthed in the sedimentary deposits; fourteen reactors were located at the Oklo-Okelobondo area and one reactor at Bangombé 30Km away [1-3].

The occurrence of these reactors has several important implications; it could be used to find precise limits on possible changes of fundamental constants, on other hand provided a model on how to retain nuclear wastes [3,4].

The operation of the Oklo reactors depends upon the existence of conditions suitable to sustain fission. Parametric studies of nuclear criticality at Oklo [5] have been carried out by R.Naudet whose can explain the criticality conditions only for the biggest reactor RZ2,

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but he can not extend this study to explain the occurrence of criticality in the smallest reactor RZ9. Using the Monte-Carlo MCNP code, the occurrence of criticality in such reactor could be explained [6,7].

In this work, an analytical method was used to study the neutron multiplication factor as function of porosity and uranium concentration in order to find the minimum size necessary for criticality in RZ9 reactor.

2. Theory

2.1. Tow group neutron diffusion theory:

Several techniques are available to compute the effective multiplication factor. An analytical method based on the two group neutron diffusion theory was used.

We consider firstly a cylindrical geometry for the RZ9 reactor with radius R and thickness H without reflector. This geometry is chosen according to the observation made in the field [2,6,10].

![Fig. 1. Geometrical model of the reactor.](image)

The tow group equations describing the reactor are [8]:

\[
\begin{align*}
D_\text{th} \Delta \Phi_\text{th} - \sum_a \Phi_\text{th} + P \sum_F \Phi_F &= 0 \\
D_\text{F} \Delta \Phi_\text{F} - \sum_F \Phi_F + \frac{K_{\infty}}{\tau K_\text{eff}} \sum_a \Phi_\text{th} &= 0
\end{align*}
\] (1)

All group fluxes have the same spatial dependence which are determined by the one group reactor equation, so, we can represent the spatial dependence of the flux with Helmoltz equations [8]:

\[
\begin{align*}
\Delta \Phi_\text{th} + B^2 \Phi_\text{th} &= 0 \\
\Delta \Phi_\text{F} + B^2 \Phi_\text{th} &= 0
\end{align*}
\] (2)

Using these equations to replace \( \Delta \) terms in Eq. (1) we obtain:

\[
K_{\text{eff}} = \frac{K_{\infty}}{(1 + B^2 M^2)}
\]

The effective multiplication factor is expressed in terms of the infinite multiplication factor and the non leakage probability for both thermal and fast neutrons. The quantities \( B^2 L^2 \) and \( B^2 \tau \) are therefor small, so, when the denominator of the critical Eq. (3) is multiplied out the term \( B^4 L^2 \tau \) can be ignored, the resulting expression is [8]:

\[
K_{\text{eff}} = \frac{K_{\infty}}{1 + B^2 M^2}
\]

Where:

\( M^2 = L^2 + \tau \), the migration area.

\( L^2 = \frac{D}{\Sigma a} \), the thermal diffusion area.

\( \tau = \frac{\sum_{i=1}^{3} \xi_{avg}}{\sum_{i=1}^{3} \xi_{avg}} \ln \left( \frac{E_i}{E} \right) \), neutron age.
The infinite multiplication factor is given by the four factor formula: 
\[ K_\infty = \eta \varepsilon P_f \]

The infinite multiplication factor is given by the four factor formula: 
\[ f = \frac{\sum a_{(fuel)}}{\sum a_{(total)}} \]
the thermalisation utilization factor 
\[ \eta = \nu \sum f_{(fuel)} \]
the thermal fission factor 
\[ \varepsilon = 1 \]
fast fission factor 

\[ P = \exp - \left( \frac{N_{eff} I_{eff}}{\sum_{a=1}^{A} \sum_{f=1}^{F} f_{(fuel)}} \right) \]
the resonance escape probability 

To compute the parameters \( K_\infty \) and \( M^2 \), the material composition of the reactor must be given.

2.2. Composition of the Oklo RZ9

The ore sample used in this study is defined by two volumes [6]: a solid volume and a fluid volume. The solid volume is divided in two volumes, uraninite and gangue (90% silica and 10% clay) which is defined according to the chemical analyses performed on various samples from the RZ9 core [6]. The element composition of the clay is given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. The element composition of the clay.</th>
</tr>
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<tbody>
<tr>
<td><strong>Chlorite</strong></td>
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<tr>
<td>Atom/Mesh</td>
</tr>
<tr>
<td>Si</td>
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<tr>
<td>Al</td>
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</tbody>
</table>

The volume fraction of uraninite is designed by \( V_{UO_2} \), it represents the volume of the uraninite in a volume of 1cm³ of the hydrated ore. The fluid used in this study is only water, it used as moderator under Oklo \((P,T)\) conditions. Its volume fraction is designed by \( \Phi_T \). So, for each values of \( V_{UO_2} \) and \( \Phi_T \) we can write the total hydrated ore density as [6]:

\[ \rho_{ore} = V_{UO_2} \* \rho_{UO_2} + \Phi_T \* \rho_{fluid} + (1 - \Phi_T - V_{UO_2}) \* \rho_{gangue} \]  (3)

2.3. Compute the macroscopic cross section

To compute the macroscopic cross section \( \Sigma = N \sigma \), we must firstly compute the number density, it is related to the density \( \rho \) and atomic number \( A \) by : \( N = N_A \rho \) where \( N_A = 6.023 \times 10^{23} \) is Avogadro’s number. For a mixture of elements the number density is:

\[ N_x = N_A f_x / A \]  (4)

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where we define the mass fraction for an ingredient $x$ as:

$$f_x = \frac{\%_{vol} x}{\rho_x}$$  \hspace{1cm} (5)

the mass fraction of uraninite, water and gangue are:

$$f_{UO_2} = V_{UO_2} \times \rho_{UO_2}$$
$$f_{\text{water}} = \Phi_T \times \rho_{\text{water}}$$
$$f_{\text{gangue}} = (1 - \Phi_T - V_{UO_2}) \times \rho_{\text{gangue}}$$  \hspace{1cm} (6)

The density of the gangue is defined as [6]:

$$\frac{1}{\rho_{\text{gangue}}} = \frac{\%_{\text{mass clay}}}{\rho_{\text{clay}}} + \frac{\%_{\text{mass silica}}}{\rho_{\text{silica}}}$$  \hspace{1cm} (7)

With:

$$\frac{1}{\rho_{\text{clay}}} = \frac{\%_{\text{mass illite}}}{\rho_{\text{illite}}} + \frac{\%_{\text{mass chlorite}}}{\rho_{\text{chlorite}}}$$  \hspace{1cm} (8)

3. Results

The aim of this work was to study the criticality of RZ9 using an analytical method. The use of Eq. (4) allowed us to study $K_{eff}$ as function of radius $R$, by varying the thickness $H$ from $60\,\text{cm}$ to $100\,\text{cm}$ by $10\,\text{cm}$ steps, under different physical conditions of $\Phi_T$ and $V_{UO_2}$, these conditions were chosen for $\Phi_T = 20\%$, $30\%$ and $40\%$ and $V_{UO_2}$ from $4\%$ to $7\%$. The radius $R$ had a $20\,\text{cm}$ variation steps.

The results obtained are shown in figures, a comparison between these results and those obtained using the MCNP code shows a satisfactory agreement.

![Graph showing effective multiplication factor versus radius of reactor for $\Phi_T = 20\%$ and $V_{UO_2} = 4\%$.](image)

**Fig. 2.** Effective multiplication factor versus radius of reactor for $\Phi_T = 20\%$ and $V_{UO_2} = 4\%$. 
This study shows us once again that the criticality may occur at low concentration and low porosity but with dimensions quite big (Fig. 2). We observe also that the critical dimensions were sensitive to porosity, they decreased from \((H = 90cm \text{ and } R = 180cm \text{ at } \Phi_T = 20\%)\) to \((H = 80cm \text{ and } R = 120cm \text{ at } \Phi_T = 40\%)\). The increase of porosity led to smaller dimensions due to the thermalisation effect.

Fig. 3. effective multiplication factor versus radius of reactor for \(\Phi_T = 30\%\) and \(V_{UO2} = 4\%\).

Fig. 4. effective multiplication factor versus radius of reactor for \(\Phi_T = 40\%\) and \(V_{UO2} = 4\%\).

Fig. 5. effective multiplication factor versus radius of reactor for different values of \(V_{UO2}\) at \(\Phi_T = 30\%\).
The Fig. (5) shows that the subcritical state with $H = 70\ cm$ at $\Phi_T = 30\%$ can achieve the criticality when $V_{UO_2}$ exceeds $4\%$. It was possible to found criticality under this conditions, the critical radius obtained is $R = 100\ cm$ ($V_{UO_2} = 5\%$), $R = 80\ cm$ ($V_{UO_2} = 6\%$) and $R = 65\ cm$ ($V_{UO_2} = 7\%$).

4. Conclusion

Using the chemical composition of the initial ore and based on the two group neutron diffusion equation, the criticality on the RZ9 reactor is studied. This analytical study allows us to find relatively the same results obtained using the MCNP code.

The aim of the continued research will be to use this equation to study the reactor with reflectors, and verify the efficacy of this method to predict the conditions that led to the operation of the RZ9 reactor.

REFERENCES